Ad Hoc And Sensor Networks
Exercise 8

Assigned: November 9, 2009
Due: November 16, 2009

1 Dominating Sets in Unit Disk Graphs

You have seen different algorithms in the lecture that come up with a Dominating Set (DS) of nodes given an input topology. Thereby a wide range of network models is assumed. In the following you are asked to investigate two DS algorithms presented in the lecture under different network models.

a) The greedy algorithm to compute a DS in arbitrary graphs was discussed in the lecture. It can be shown that the algorithm is $\log \Delta$ approximation in this model. What is the approximation ratio for the greedy algorithm for the UDG model?

b) You have also seen the Grid Algorithm to compute a DS. In this algorithm a virtual grid with a cell diagonal of 1 (or the transmission range) is laid over the physical network. Within each cell the node closest to the cell center becomes the dominator for all other nodes in the cell. Can you adopt this algorithm to produce a connected dominating set (CDS) by decreasing the size of the cells? Give a bound on the size of the cell where a CDS is constructed or argue why it is not possible.

Connectivity Models

In the lecture you have learned that researchers have studied different models for wireless sensor networks. Such models are often simplifications of reality but allow to give formal proofs on the correctness or performance of an algorithm.

a) You are asked to model the connectivity of two different sensor networks: The first sensor network is used by a farmer in the outback of Australia to monitor his cattle; the second network is located inside an office building. Which model would you recommend and why?

b) Given the connectivity set $E = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_2, v_5\}, \{v_4, v_5\}, \{v_5, v_6\}, \{v_5, v_7\}, \{v_6, v_7\}\}$, consider the following node positions in the 2-dimensional Euclidean plane: $V = \{v_1 = (1, 2), v_2 = (2, 1), v_3 = (2, 3), v_4 = (3, 2), v_5 = (5, 5), v_6 = (2, 4), v_7 = (2, 5)\}$. Is the graph $G(V, E)$ a Unit Disk Graph?

c) Can graph $G(V, E)$ presented in b) be modeled as a Quasi Unit Disk Graph? If yes, what is the maximal ρ?

d) If only the connectivity information E is considered but not the absolute positions, can the resulting graph be modeled as a Unit Disk Graph?

e) Can E be modeled as a Bounded Independence Graph? Find the nodes for which the 1-hop neighborhood contains the largest set of independent nodes; what is the cardinality of this set? What is the solution for the case of the 2-hop neighborhood?

f) Can E be modeled as a Unit Ball Graph? Define a metric on the nodes of E which maintains E's connectivity and which has a minimal doubling dimension.

g) Formally show that there exist graphs which fulfill the metric space property but which do not have a constant doubling dimension.