Chapter 7

MAC Theory

7.1 Slide 7/17

Definition 7.1. An event happens “with high probability” (w.h.p.) if it happens with probability at least $1 - 1/n^c$, for some arbitrary constant $c$.

Theorem 7.2. If nodes wake up in an arbitrary (worst-case) way, any algorithm may take $\Omega(n/\log n)$ time slots until a single node can successfully transmit.

Proof. Nodes must transmit at some point, or they will surely never successfully transmit. With a uniform protocol, every node executes the same code. We focus on the first slot where nodes may transmit. No matter what the protocol is, this happens with probability $p$. Since the protocol is uniform, $p$ must be a constant, independent of $n$.

The adversary wakes up $w := \frac{c}{\ln n}$ nodes in each time slot, with some constant $c$. All nodes woken up in the first time slot will transmit with probability $p$. We study the event $E_1$ that exactly one of them transmits in that first transmission slot. Using the inequality $(1 + t/n)^n \leq e^t$ we get

$$P[\text{Pr}[E_1] = w \cdot p \cdot (1 - p)^{w-1} = c \ln n \cdot (1 - p)^{\frac{1}{2}(c \ln n - p)} \leq c \ln n \cdot e^{-c \ln n + p} = c \ln n \cdot n^{-c} \cdot e^p = n^{-c} \cdot O(\log n) < \frac{1}{n^{c-1}} = \frac{1}{n^{c_2}}.$$  

In other words, w.h.p. that slot will not be successful. Let $E_a$ be the event that all $n/w$ slots will not be successful. Using the inequality $1 - p \leq (1 - p/k)^k$ we get

$$P\left[\text{Pr}[E_a] = (1 - \text{Pr}[E_1])^{n/w} \geq \left(1 - \frac{1}{n^{c_2}}\right)^{O(n/\log n)} \right] > 1 - \frac{1}{n^{c_3}}.$$  

In other words, w.h.p. it takes more than $n/w$ slots until some node can transmit alone.  

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