Ad Hoc And Sensor Networks
Sample Solution to Exercise 12

Assigned: December 13, 2010
Due: December 20, 2010

1 MAC Layer with Power Control

a) We have to check whether the Signal-to-Noise-plus-Interference ratio (SINR) is at least 2 for nodes C and D. The SINR for Node C and D are:

$$\text{SINR}_C = \frac{\frac{P_B}{d(b,c)^\alpha}}{N + \frac{P_A}{d(a,c)^\alpha}} = 113.64 \tag{1}$$

$$\text{SINR}_D = \frac{\frac{P_A}{d(a,d)^\alpha}}{N + \frac{P_B}{d(b,d)^\alpha}} = 0.077 \tag{2}$$

Therefore, Node C is able to receive packets sent by Node B while the received signal from Node A is too weak to be correctly decoded by Node D.

b) If Node B is not sending, the SINR at Node D depends only on the transmission power P_A and the channel noise N:

$$\text{SINR}_D = \frac{\frac{P_A}{d(a,d)^\alpha}}{N} = 3.64 \tag{3}$$

Thus, Node D can receive Node A's signal if no interference from Node B is present. We now calculate what the maximal transmission power of Node B can be such that the SINR at Node D is still above 2.

$$\text{SINR}_D = \frac{\frac{P_A}{d(a,d)^\alpha}}{N + \frac{P_B}{d(b,d)^\alpha}} \geq 2 \Longrightarrow P_B \leq 0.178\text{mW} \tag{4}$$

On the other hand, in order to have a SINR ≥ 2 at Node C, the minimal transmission power P_B has to be at least 0.176 mW:

$$\text{SINR}_C = \frac{\frac{P_B}{d(b,c)^\alpha}}{N + \frac{P_A}{d(a,c)^\alpha}} \geq 2 \Longrightarrow P_B \geq 0.176\text{mW} \tag{5}$$

Therefore, the minimal necessary transmission power for Node B is set to $P_B = 0.176\text{mW}$.

c) The overall power consumption is minimized if both transmitters are sending with a transmission power that leads to a SINR of exactly 2 at both receivers. Thus, the transmission power values are obtained by solving the following equation for P_A and P_B:

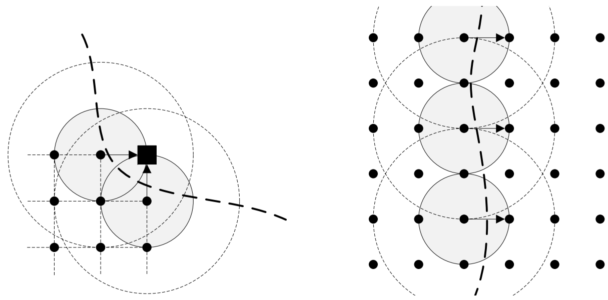
$$\frac{\frac{P_B}{d(b,c)^\alpha}}{N + \frac{P_A}{d(a,c)^\alpha}} = \frac{\frac{P_A}{d(a,d)^\alpha}}{N + \frac{P_B}{d(b,d)^\alpha}} = 2 \tag{6}$$

The resulting transmission power values are $P_A = 9.932\text{mW}(22.96\text{dBm})$ for Node A and $P_B = 0.175\text{mW}(-17.43\text{dBm})$ for Node B.

2 Capacity Limits in Sensor Networks

a) The max-flow min-cut theorem states that the maximum traffic flow is equal to the capacity of the minimum cut. In other words, the maximum flow in the given sensor network is dictated by its bottleneck. Obviously, the bottleneck of this sensor network is the sink node since all sensor reading have to be forwarded eventually to the sink node. The cut around the sink is indicated by the strong dashed line in Figure 1(a). Given the one-hop communication range of the sensor nodes, indicated as the filled circle in the figure, the cut contains two edges between the sink node and its two adjacent nodes. This leads to a capacity of W bit/s for this cut. Therefore, the maximum capacity of this sensor network is W bit/s.

b) We use again a cut that divides the network into two parts, see Figure 1(b). Given that traffic flows between randomly chosen source and destination pairs, on average half of the traffic has to flow through the edges of this cut leading to the bottleneck of this network. Therefore, we are interested in the maximum capacity of these edges. In other words, we want to know how many of the nodes can simultaneously send data through the edges of the cut. Two nodes can send at the same time if their receiver nodes are not in the interference range of the other node. The communication range of a node is depicted with a filled circle and the interference range of a node is indicated with a dashed circle in Figure 1(b). As we can see from the picture, only every second node can send at the same time leading to a maximum flow of $\frac{W \cdot \sqrt{n}}{2}$ through the cut. Therefore, the capacity of the sensor network is $\Theta(W \cdot \sqrt{n})$ according to the max-flow min-cut theorem.



(a) (b)

Figure 1: Network capacity for a grid with a base station (left) or random traffic (right).