



# Ad Hoc And Sensor Networks

## Sample Solution to Exercise 9

Assigned: November 22, 2010

Due: November 29, 2010

### 1 Accoustic Positioning

- a) If all clocks run at the nominal frequency of 32 kHz and are perfectly synchronized, the accuracy of the measurement is only limited by the clock granularity. A single clock tick has a duration of exactly  $\frac{1}{32768}$  s. During this time a sound wave ( $c \approx 343$  m/s) travels approximately 0.0105 m. Therefore, the system can distinguish two positions if they are more than 1cm apart.
- b) The clock used on the sensor nodes drifts up to 50 ppm<sup>1</sup> from the nominal frequency of 32 kHz. Therefore, a clock can differ up to 48 ticks during the synchronization interval of 30 seconds from a clock running at the nominal frequency of 32 kHz. The measured propagation delay between the marmot and a sensor node can differ up to 48 clock ticks from the real value. Thus, the position estimated by the measurement of a single sensor node is incorrect up to  $\approx 51$  cm as indicated by the arrows in Figure 1. Consequently, the estimated position is within a circle of radius  $r = 51$  cm around the real position of the marmot.

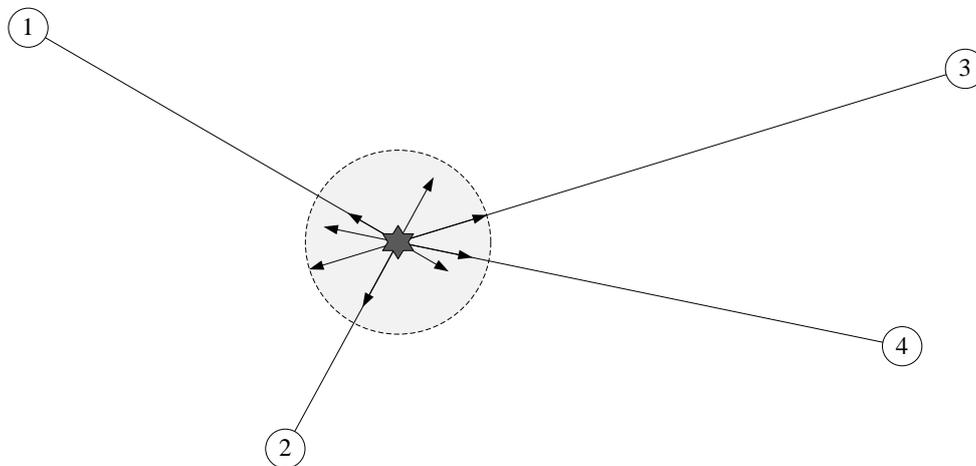


Figure 1: The real position of the marmot is depicted by the star. The uncertainty in the estimation by the sensor nodes due to clock drift is indicated by the arrows.

- c) A simple solution would be to replace the hardware clocks with more accurate ones. Since more precise clocks are also more expensive, one can also mitigate the effects of drifting clocks by drift compensation in software. Both the local clock value and the clock value of the reference node are stored in a table for each synchronization point. Since it is assumed that the clock drift is relatively stable for a short time period, linear regression can be used to find the best-fit line between the local and the reference clock. The best-fit line can be used to estimate the reference clock value given the local hardware clock reading.

<sup>1</sup>ppm = parts per million. A clock having 50 ppm drift from the nominal frequency of 1 MHz will be increased  $10^6 \pm 50$  clock ticks during a second.

## 2 Spring Embedding

- a) The simple spring embedder computes the resulting force  $F_i$  for all nodes  $i \in V$ . In our case this is only node 2. We see that no matter where that starting point of node 2 lies, the other two nodes effect the same force on it. Given an attenuation factor of 1 the spring embedder overshoots the steady state, that is position  $(0,1)$ , by the same amount in the first iteration. So, the position of node 2 is  $(0,1.4)$ . It can be seen that the system is not converging but that node 2 is oscillating between these two positions.
- b) If  $\rho$  is set to 0.9 the node is getting closer to  $(0,1)$  with each iteration of the spring embedder. However, It will never reach  $(0,1)$ . One can see that the ideal attenuation factor is 0.5 as the system stabilizes already after the first iteration.