Eidgenössische Technische Hochschule Zürich
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Computer Engineering and Networks Laboratory

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Discrete Event Systems

Exercise Sheet 13

1 Computation Tree Logic Model Checking

Let the Kripke structure $\mathcal{K}$ be defined as follows.

$$\mathcal{K} := \left\{ \begin{array}{l} \text{States} := \{1,2,3,4\} \\ \mathbb{S}_0 := \{s_0\} = \{1\} \\ \rightarrow := \{(1,3),\ (3,2),\ (2,1),\ (2,4),\ (4,2)\} \\ AP := \{\text{green}, \text{yellow}, \text{red}, \text{black}\} \\ L := \{1 \mapsto \text{red},\ 2 \mapsto \text{yellow},\ 3 \mapsto \text{green},\ 4 \mapsto \text{black}\} \end{array} \right\}$$

Furthermore, we define the following CTL formulas:

$\Omega_1 = \exists \Box \text{green}$

$\Omega_2 = \forall \Box \text{yellow}$

$\Omega_3 = \exists (\text{yellow} \Rightarrow (\forall \bigcirc \text{black}))$

$\Omega_4 = \forall \Diamond \text{black}$

$\Omega_5 = \exists \bigcirc (\text{true} \bigcup \text{black})$

$\Omega_6 = \forall (\text{black} \bigcup \text{black})$

$\Omega_7 = \forall (\neg \text{yellow} \bigcup (\exists \bigcirc \text{black}))$

$\Omega_8 = \exists (\text{black} \bigcup \text{black})$

a) Draw the graph for the Kripke structure $\mathcal{K}$.

b) Give the computation tree for the initial state s_0 upto depth 7 (root node has depth 1).

c) Which of the above CTL formulas are syntactically incorrect? Justify your answers.

d) Transform the syntactically correct CTL formulas into existential normal form (ENF).

2 Petri Nets [Exam!]

In this exercise you are supposed to model a function $f_i(x, y)$ on a petri net. That is, the petri net must contain two places P_x and P_y that hold x and y tokens respectively in the beginning. Additionally, the net must contain one place P_z which holds $f_i(x, y)$ tokens when the net is dead. The petri nets are supposed to work for arbitrary numbers of tokens in P_x and P_y.

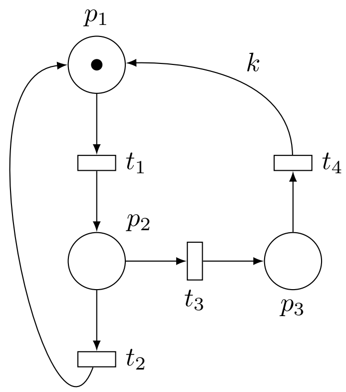
a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$

b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$

c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$

3 Basic Properties of Petri Nets

Given is the following petri net N_2:



a) Explain the terms *boundedness* and *deadlock-freeness* using this example.

b) For which values of $k \in \mathbb{N}$ is the petri net N_2 bounded/unbounded and not deadlock-free?

4 Reachability Analysis for Petri Nets

In the lecture we presented an algorithm to perform a reachability analysis on petri nets.

a) Why is it not possible with a reachability algorithm to determine *in general*, whether a given state in a petri net is reachable or not?

b) Consider the petri net N_2 from exercise 3. Is the state $s = (p_1 = 101, p_2 = 99, p_3 = 4)$ reachable from the initial state $s_0 = (1, 0, 0)$ if $k = 2$? Prove your answer.

Note: Use the condition presented in the lecture that is sufficient for the reachability of a state in a weighted petri net.