Discrete Event Systems  
Exercise Sheet 13

1 Computation Tree Logic Model Checking

Let the Kripke structure $\mathcal{K}$ be defined as follows.

\[
\begin{align*}
\mathcal{K} := \left\{ 
\begin{array}{l}
\text{States := \{1, 2, 3, 4\}} \\
S_0 := \{s_0\} = \{1\} \\
\rightarrow := \{(1, 3), (3, 2), (2, 1), (2, 4), (4, 2)\} \\
AP := \{\text{green, yellow, red, black}\} \\
L := \{1 \mapsto \text{red}, 2 \mapsto \text{yellow}, 3 \mapsto \text{green}, 4 \mapsto \text{black}\}
\end{array}
\right. 
\end{align*}
\]

Furthermore, we define the following CTL formulas:

\[
\begin{align*}
\Omega_1 &= \exists \square \text{green} \\
\Omega_2 &= \forall \square \text{yellow} \\
\Omega_3 &= \exists (\text{yellow} \Rightarrow (\forall \bigcirc \text{black})) \\
\Omega_4 &= \forall \lozenge \text{black} \\
\Omega_5 &= \exists \bigcirc (\text{true} \bigcup \text{black}) \\
\Omega_6 &= \forall (\text{black} \bigcup \text{black}) \\
\Omega_7 &= \forall (\neg \text{yellow} \bigcup (\exists \bigcirc \text{black})) \\
\Omega_8 &= \exists (\text{black} \bigcup \text{black})
\end{align*}
\]

a) Draw the graph for the Kripke structure $\mathcal{K}$.

b) Give the computation tree for the initial state $s_0$ up to depth 7 (root node has depth 1).

c) Which of the above CTL formulas are syntactically incorrect? Justify your answers.

d) Transform the syntactically correct CTL formulas into existential normal form (ENF).

2 Petri Nets [Exam!]

In this exercise you are supposed to model a function $f_i(x, y)$ on a petri net. That is, the petri net must contain two places $P_x$ and $P_y$ that hold $x$ and $y$ tokens respectively in the beginning. Additionally, the net must contain one place $P_z$ which holds $f_i(x, y)$ tokens when the net is dead. The petri nets are supposed to work for arbitrary numbers of tokens in $P_x$ and $P_y$.

a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$

b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$

c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$
3 Basic Properties of Petri Nets

Given is the following petri net $N_2$:

![Petri Net Diagram]

a) Explain the terms *boundedness* and *deadlock-freeness* using this example.

b) For which values of $k \in \mathbb{N}$ is the petri net $N_2$ bounded/unbounded and not deadlock-free?

4 Reachability Analysis for Petri Nets

In the lecture we presented an algorithm to perform a reachability analysis on petri nets.

a) Why is it not possible with a reachability algorithm to determine in general, whether a given state in a petri net is reachable or not?

b) Consider the petri net $N_2$ from exercise 3. Is the state $s = (p_1 = 101, p_2 = 99, p_3 = 4)$ reachable from the initial state $s_0 = (1, 0, 0)$ if $k = 2$? Prove your answer.

*Note:* Use the condition presented in the lecture that is sufficient for the reachability of a state in a weighted petri net.