Eidgenössische Technische Hochschule Zürich
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Distributed Computing

HS 2010

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Discrete Event Systems

Exercise Sheet 7

1 Probability of Arrival

In the script, there is a lemma saying that the probability of arrival can be computed as

$$f_{ij} = p_{ij} + \sum_{k:k \neq j} p_{ik} f_{kj} \quad .$$

Prove this lemma.

2 Basketball [Exam]

Mario, Luigi and Trudy meet to play basketball. To improve their scoring abilities, Mario suggests the following game: Each player has to score m times. After each miss, he has to perform 10 push-ups.

a) Assume that Mario always scores with a constant probability p. How many push-ups does he do in expectation in his game?

b) Luigi wants to show that he is better and wants to score m times in sequence. After each miss, he performs 10 push-ups as well, and then tries again to score m times in a row.

(i) How many push-ups does Luigi in expectation, assuming he also scores with a constant probability p?

(ii) How many shots will Luigi do in expectation? (Tricky bonus exercise)
Hint: Show first that for the number of shots S in an unsuccesful round, we have $\mathbf{E}[S] \leq \frac{1}{1-p}$ and then use this fact.

c) Trudy accepts Luigi's game and tries to score m times in a row. But Trudy is a bit lazy and gives up as soon as she has missed two times in a row. Trudy scores with constant probability $p = 0.5$, and for her we know that $m = 3$.

(i) What is the probability that Trudy scores $m = 3$ times in a row? What is the probability that she gives up?

(ii) How many push-ups does Trudy do in expectation?

Hint:
$$\sum_{i=1}^{\infty} i \cdot q^{i-1} = \frac{1}{(1-q)^2} \qquad \text{for } |q| < 1$$

3 Night Watch [Exam]

In order to improve their poor financial situation, Jasmin and Tobias also work at nights. Their task is to guard a famous Swiss bank which, from an architectonic perspective, looks as follows:

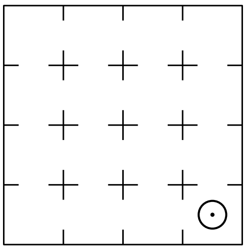

Figure 1: Offices of a Swiss bank.

Thus, there are 4x4 rooms, all connected by doors as indicated in the figure.

In a first scenario, Tobias and Jasmin always stay together. They start in the room on the upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.

a) Compute the probability (in the steady state) that Jasmin and Tobias are in the room where the thief enters the bank (indicated with $\odot$)!

b) Since Tobias and Jasmin are very strong, they can easily catch a thief on their own. Thus, in a second scenario, they decide that it would be smarter to patrol individually: After every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?