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Discrete Event Systems

Solution to Exercise Sheet 12

1 Labelled Graphs

a) We give two algorithms, an iterative and a recursive one, that calculate whether the given LTS $\mathcal{L}$ accepts the word $\omega = w_1 \dots w_n$.

Algorithm 1 AcceptIterative($\mathcal{L}, \omega$)

```
Input: LTS L = (S, S_0, Act, E), ω = w_1 w_2 ... w_n
states ← S_0                                        ▷ contains the states reachable by w_1 ... w_{i-1}
for i ← 1 to n do
    newStates ← ∅                                   ▷ contains the new states reachable by w_1 ... w_i
    for all v ∈ states do                           ▷ For all current states...
        for all c ∈ PostSetNodes(v) do              ▷ For all reachable states...
            if Act((v, c)) = w_i then               ▷ If the label matches...
                newStates ← newStates ∪ {e.target()}  ▷ ...remember the state
    if newStates = ∅ then                           ▷ If no edge with label w_i exists...
        return false
    states ← newStates
return true
```

Algorithm 2 AcceptRecursive($\mathcal{L}$, states, ω)

```
Input: LTS L = (S, S_0, Act, E), states: set of states, ω = w_1 w_2 ... w_n
newStates ← ∅
if ω = ∅ then                                       ▷ Every letter of the word has been matched to a path.
    return true
else if states = ∅ then                             ▷ No state was reachable by the last letter.
    return false
for all v ∈ states do                               ▷ For all current states...
    for all c ∈ PostSetNodes(v) do                  ▷ For all reachable states...
        if Act((v, c)) = w_1 then                   ▷ If the label matches...
            newStates ← newStates ∪ {c}             ▷ ...remember the state
ω ← w_2, ..., w_n                                   ▷ Remove first letter of ω
return AcceptRecursive(L, newStates, ω)             ▷ Recursive call for the remaining word
```

The initial call is AcceptRecursive($\mathcal{L}, \mathbb{S}_0, \omega$).

2 Structural Properties of Petri Nets and Token Game

a) The pre and post sets of a transition are defined as follows:

- pre set: $\bullet t := \{p \mid (p,t) \in C\}$
- post set: $t\bullet := \{p \mid (t,p) \in C\}$,

the pre and post sets of a place are defined analogously.

For the petri net N_1 we obtain the following sets:

$$\begin{aligned} \bullet t_5 &= \{p_5, p_9\}, & t_5\bullet &= \{p_6\} \\ \bullet t_8 &= \{p_8\}, & t_8\bullet &= \{p_{10}, p_5\} \\ \bullet p_3 &= \{t_2\}, & p_3\bullet &= \{t_3\} \end{aligned}$$

b) A transition is enabled if all places in its pre set contain enough tokens. In the case of N_1, which has only unweighted edges, one token per place suffices. When t_2 fires, it consumes one token out of each place in the pre set of t_2 and produces one token on each place in the post set of t_2. Hence, the firing of t_2 produces one token on place p_3 and p_9 each, the one on p_2 is consumed. After this, t_5 is enabled because both p_9 and p_5 hold one token. However, t_3 is not enabled because p_3 contains a token but p_{10} does not.

c) Before t_2 fires there are two tokens in N_1, one on p_2 and p_5 each. Directly afterwards, there are tokens on places p_3, p_9 und p_5.

d) A token traverses the upper cycle until t_2 fires. Then one token remains on p_3 and waits, and another one is produced in p_9, which enables transition t_5. When t_5 consumes the tokens on p_9 and p_5 and produces a token on p_6, this one can traverse the lower cycle until t_8 is enabled. One token now remains on p_5 and waits, another one enables t_3, because there is still one token on p_3. Now one token traverses the upper cycle again until t_2 is enabled, and so on.

Hence, this petri net models two processes which always appear alternately.

The reachability graph $RG(P, \vec{s}_0)$ of a petri net P is a quadruple $(\mathbb{S}, \mathbb{S}_0, Act, \mathbb{E})$ such that

- $\mathbb{S}$ is the set of reachable states of P starting from $\vec{s}_0$
- $\mathbb{S}_0 := \{\vec{s}_0\}$ is the start state of P
- Act is the set of transition labels
- $\mathbb{E} \subseteq \mathbb{S} \times Act \times \mathbb{S}$ is the set of edges such that $\mathbb{E} = \{(\vec{s}, t, \delta(\vec{s}, t)) \mid \vec{s} \in \mathbb{S} \wedge t \in T \wedge \vec{s} \triangleright t\}$

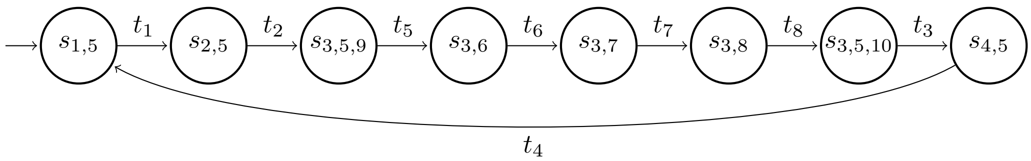
Usually the states of the petri net are denoted by vectors such that the i-th position in the vector indicates the number of tokens on place p_i of the petri net. So, for example, the starting state $\vec{s}_0$ of N_1, in which the places p_1 and p_5 hold one token each, is denoted by

$\vec{s}_0 = (1,0,0,0,1,0,0,0,0,0)$. Hence, the reachability graph looks as follows:

$$\begin{aligned}
\mathbb{S} = \{ \quad & (1,0,0,0,1,0,0,0,0,0),(0,1,0,0,1,0,0,0,0,0),(0,0,1,0,1,0,0,0,1,0),\\
& (0,0,1,0,0,1,0,0,0,0),(0,0,1,0,0,0,1,0,0,0),(0,0,1,0,0,0,0,1,0,0),\\
& (0,0,1,0,1,0,0,0,0,1),(0,0,0,1,1,0,0,0,0,0) \quad \},\\
\mathbb{S}_0 = \{ \quad & (1,0,0,0,1,0,0,0,0,0) \quad \},\\
Act = \{ \quad & t_1,t_2,t_3,t_4,t_5,t_6,t_7,t_8,t_9,t_{10} \quad \},\\
\mathbb{E} = \{ \quad & \big((1,0,0,0,1,0,0,0,0,0),t_1,(0,1,0,0,1,0,0,0,0,0)\big),\\
& \big((0,1,0,0,1,0,0,0,0,0),t_2,(0,0,1,0,1,0,0,0,1,0)\big),\\
& \big((0,0,1,0,1,0,0,0,1,0),t_5,(0,0,1,0,0,1,0,0,0,0)\big),\\
& \big((0,0,1,0,0,1,0,0,0,0),t_6,(0,0,1,0,0,0,1,0,0,0)\big),\\
& \big((0,0,1,0,0,0,1,0,0,0),t_7,(0,0,1,0,0,0,0,1,0,0)\big),\\
& \big((0,0,1,0,0,0,0,1,0,0),t_8,(0,0,1,0,1,0,0,0,0,1)\big),\\
& \big((0,0,1,0,1,0,0,0,0,1),t_3,(0,0,0,1,1,0,0,0,0,0)\big),\\
& \big((0,0,0,1,1,0,0,0,0,0),t_4,(1,0,0,0,1,0,0,0,0,0)\big) \quad \}.
\end{aligned}$$

For better legibility we denote the states in such a way that the index contains the places that hold a token in this state, for example $\vec{s}_0 = (1,0,0,0,1,0,0,0,0,0) = s_{1,5}$.

Then the reachability graph can also be specified as follows:



3 Basic Properties of Petri Nets

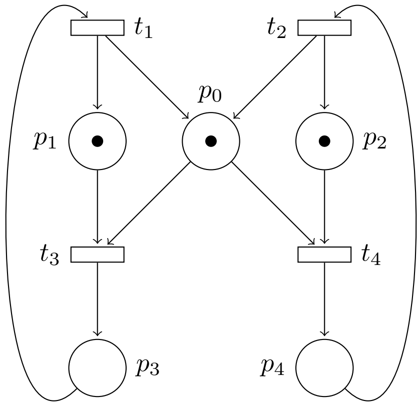
See exercise sheet 13.

4 Reachability Analysis for Petri Nets

See exercise sheet 13.

5 Mutual Exclusion

For each process we introduce two places (p_1, p_2, p_3 und p_4) representing the process within the normal program execution (p_1, p_2) as well as in the critical section (p_3, p_4). For each process, we have a token indicating which section of the program currently is executed. Additionally, we introduce a place p_0 representing the mutex variable. If the mutex variable is 0, then we have a token at p_0. We have to make sure that a process can only enter its critical section if there is a token at the mutex place. The resulting petri net looks as follows.



Assume that initially, both processes are in an uncritical section. A process can only enter its critical section if there is a token at p_0. In thise case, the token is consumed when entering the critical section. A new mutex token at p_0 is not created until the process leaves its critical section. Hence, both processes exclude each other mutually from the concurrent access to the critical section.