Discrete Event Systems
Solution to Exercise Sheet 13

1 Computation Tree Logic Model Checking

a) The graph of the Kripke structure $K$ looks as follows:

```
1
(red)

2
(yellow)

3
(green)

4
(black)
```

b) The computation tree for the initial state $s_0$ upto depth 7 looks as follows:

```
(red) 1

(green) 3

(yellow) 2

(red) 1

(green) 3

(yellow) 2

(red) 1

(black) 4

(green) 3

(yellow) 2

(red) 1

(black) 4

(green) 3

(yellow) 2

(black) 4

... ...
```
c) $\Omega_3$ is incorrect, since the existence quantor is only defined in combination with a temporal operator ($\bigcirc, \square, \bigcup, \Diamond$). Here it is used with a state formula.

$\Omega_5$ is incorrect, since the operator $\exists \bigcirc$ is only defined for a state formula. (true $\bigcup$ black), however, is a path formula.

d) See solution to exercise sheet 14.

2 Petri Nets [Exam!]

a) $f_1(x, y) = 5x + y$:

b) $f_2(x, y) = x - 2y$:

c) $f_3(x, y) = x \cdot y$:

3 Basic Properties of Petri Nets

A petri net is $k$-bounded, if there is no fire sequence that makes the number of tokens in one place grow larger than $k$. It is obvious that petri net $N_2$ is 1-bounded if $k \leq 1$. This holds because in the initial state there is only one token in the net, and in the case $k \leq 1$ no transition increases the number of tokens in $N_2$. If $k \geq 2$, the number of tokens in $p_1$ can grow infinitely large by repeatedly firing $t_1$, $t_3$ and $t_4$. So, the petri net $N_2$ is unbounded for $k \geq 2$.

A petri net is deadlock free if no fire sequence leads to a state in which no transition is enabled. If $k = 0$, $N_2$ is not deadlock-free. The fire sequence $t_1, t_3, t_4$ causes the only existing
token to be consumed and hence, there is no enabled transition any more. For $k \geq 1$, however, no deadlock can occur.

4 Reachability Analysis for Petri Nets

a) Petri nets may possess infinite reachability graphs, e.g. $N_2$ with $k \geq 2$. If the state in question is actually reachable in such a petri net, the reachability algorithm will eventually terminate. If it is not reachable, the algorithm will never be able to determine this with absolute certainty (cf. halting problem).

b) We determine the incidence matrix of the petri net as explained in the lecture.

$$A = \begin{pmatrix} -1 & 1 & 0 & 2 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

We are interested in whether the state $\vec{s} = (101, 99, 4)$ is reachable from the initial state $\vec{s}_0 = (1, 0, 0)$. If the equation system $A \cdot \vec{f} = \vec{s} - \vec{s}_0$ has no solution, we know that the state $\vec{s}$ is not reachable from $s_0$. “Unfortunately”,

$$\begin{pmatrix} -1 & 1 & 0 & 2 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} 100 \\ 99 \\ 4 \end{pmatrix}$$

is satisfiable. To show that $\vec{s}$ is reachable from $\vec{s}_0$, we have to give a firing sequence through which we get from $\vec{s}_0$ to $\vec{s}$. From the last equation of the above equation system, we know that $f_3 = f_4 + 4$. Hence, in the desired firing sequence, $f_3$ is fired four times more than $f_4$. However, $\vec{f}$ does not tell us about the firing order. Considering the petri net, we can see that – starting from $\vec{s}_0$ – the number of tokens in $p_1$ increases by one after firing $t_1$, $t_3$, and $t_4$ in this order. Repeating this for 203 times yields the state $(204, 0, 0)$. Firing $t_1$ for 103 times followed by firing $t_3$ for four times finally yields state $\vec{s}$. 