1 Computation Tree Logic Model Checking

a) $\Omega_1$ und $\Omega_6$ are already in ENF. The remaining formulas are transformed as follows.

- $\Omega_2 \equiv \neg \exists (\text{true} \cup \neg \text{yellow})$
- $\Omega_3 \equiv \forall (\text{true} \cup \text{black}) \equiv \neg \exists (\neg \text{black} \cup \neg \text{true}) \land \neg \exists \neg \text{black}$
- $\Omega_4 \equiv \neg \exists (\neg \text{black} \cup \neg \text{black}) \land \neg \exists \neg \text{black}$
- $\Omega_5 \equiv \neg \exists \left( \neg (\exists \circ \text{black}) \cup (\text{yellow} \land \neg (\exists \circ \text{black})) \right) \land \neg \exists \neg (\exists \circ \text{black})$

b) We first give the annotated syntax trees for the syntactically correct formulas. The calculation of the satisfiability sets is explained examplarily in f) for $\Omega_5$. For the remaining formulas, the calculation works analogously.

- $\Omega_1 = \exists \circ \text{green}$:

```
  \exists \circ
    \emptyset
  \text{green} \{3\}
```

- $\Omega_2 = \neg \exists (\text{true} \cup \neg \text{yellow})$:

```
  \neg
    \emptyset
  \exists \cup \{1, 2, 3, 4\}
    \{1, 2, 3, 4\} \text{true} \neg \{1, 3, 4\}
      \text{yellow} \{2\}
```

- $\Omega_3 = \neg \exists (\neg \text{black} \cup \neg \text{true}) \land \neg \exists \neg \text{black}$:
\[ \Omega_4 = \neg\exists(\neg\text{black} \cup \neg\text{black}) \land \neg\exists\neg\text{black} \]

\[ \Omega_5 = \neg\exists\left(\neg(\exists \circ \text{black}) \cup (\text{yellow} \land \neg(\exists \circ \text{black}))\right) \land \neg\exists\neg(\exists \circ \text{black}) \]

\[ \Omega_6 = \exists(\text{black} \cup \text{black}) \]

We explain the calculation of the satisfiability sets \( \text{Satisfy}(\phi) \) exemplarily for \( \Omega_5 \). The \( \text{Satisfy}(\phi) \)-sets are first determined for the leaves of the syntax tree and then the nodes of the tree are labelled in a bottom-up fashion.
c) A formula $\Omega$ is satisfied by $K$ if the intersection of $S_0$ and $\text{Satisfy}(K)$ at the root of the syntax tree is not empty. This is only the case for $\Omega_3$. All other formulas are not satisfiable in $K$.

d) We shall have a look at the computation tree.

A path $\Pi_1$ starting in state 1 falsifies $\Omega_2$ and $\Omega_4$ because already in state 1 we have $\neg$yellow as well as $\neg$black. The path $\Pi_2 = (1, 3, 2, 1, 3, 2, \ldots)$ falsifies $\Omega_3$ because this path never reaches state 4 and hence never fulfills black.

2 Timed Automata

Solutions can be found in the reference material for this exercise as given on the course website or at http://disco.ethz.ch/lectures/hs10/des/exercises/UppaalSolutions.zip.