Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Exercise Sheet 10

1 Competitive Analysis

In this exercise, we analyze algorithms for cellular networks such as GSM. In such networks, the area is segmented into cells, each of which containing a base station. Due to interference, base-stations in neighboring (adjacent) cells cannot use the same carrier frequencies, but frequencies may be reused in non-interfering cells (i.e. cells that are not neighboring). In this exercise, we use the idealized hexagonal grid for modelling these cells (cf. Figure 1).

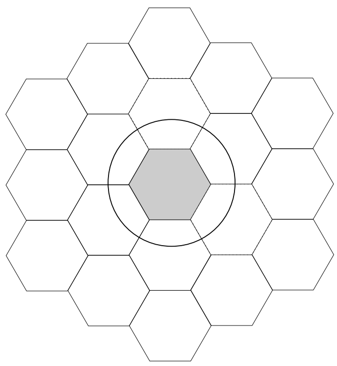

Figure 1: The hexagonal grid modelling the cells.

The number of frequencies in real GSM networks is limited. If there are more callers than channels, some calls must be rejected. In this exercise, we make the simplifying assumption that there is only a *single channel* shared by all base stations. That is, every base station can accept exactly one call. Furthermore, if a call is accepted by a base station in a certain cell, no calls can be accepted in neighboring cells due to interference.

Assume that callers arrive in an online fashion, that is, one after another in an input sequence σ. We need to accept or reject callers such that there is at most 1 caller in a cell and its 6 neighboring cells. The *benefit* of the algorithm is the number of calls we accept.

a) Assume that calls have *infinite duration*. Once a call is accepted, it remains active forever. Describe a natural greedy algorithm for the problem. Is your algorithm c-competitive for any fixed constant c? If so, what is the value of c?

b) Assume that every call can have an arbitrary duration, but base stations are not allowed to preempt accepted calls. Propose a competitive online algorithm for this scenario.

2 Power-Down Mechanisms

Since Alice has not been very happy with her internship at the processor manufacturer *Tintel*, she has applied for another internship at the company *Hewlate-Packard*, which is well-known for its energy-saving laptop models. The company's intelligence division has recently acquired a laptop from their strongest competitor *Samson* which seems to outperform their own models. Alice's first assignment is to investigate further into this matter.

The state-of-the-art laptops of both Hewlate-Packard and Samson have two power modes:

- *Active state*: Energy consumption is 1 energy unit per time unit
- *Sleep state*: Energy consumption is 0 energy unit per time unit

The transition from the active state to the sleep state and back to the active state costs D energy units altogether. Over time, there are busy periods where the user works with the device and idle periods where the laptop is not used. During busy periods, the laptop must be in the active state.

The **power-down strategy** determines the idle time after which the laptop goes to sleep. The competitiveness of a strategy S is the ratio of the energy units spent by S and the energy units spent by the optimal algorithm for a given busy sequence.

Hint: To show that a strategy is c-competitive, it suffices to show that it is c-competitive for an arbitrary idle period $I = [t_1, t_2]$. At time $t_1 - \varepsilon$, both algorithms have to be in the active state because the busy period has not ended yet. At time $t_2 + \varepsilon$, both algorithms have to be in the active state again because a busy period has just begun. Hence, it suffices to compare the energy consumption of ALG and OPT within the time span $[t_1, t_2]$ (energy costs for going to sleep state and back to active state are part of the costs for I).

a) To get familiar with power-down strategies, Alice has to find a deterministic 2-competitive power-down strategy.

b) Alice's supervisor Brian tells Alice that nobody in the company has yet found a strategy that is better than 2-competitive. Prove that no deterministic strategy can be better than 2-competitive.

c) Brian is unhappy about Alice's findings from part **b)** because he knows that the power-down strategy in Samson's laptops is better than 2-competitive. Alice has to find a strategy with a competitive ratio $c < 2$ otherwise she will be fired. Help Alice!

Hint: Alice recently heard her colleagues talking about probabilistic strategies…