1 Finite Automata and Regular Languages [Exam]

Consider the NFA $A$ in Figure 1 and further assume $\Sigma = \{0, 1\}$.

![Figure 1: NFA $A$.](image)

a) Transform the NFA into an equivalent DFA using the powerset construction of the lecture.  
   *(Hint: Only construct states which are necessary!)*

b) Give a regular expression for the language accepted by the automaton $A$?

2 Pumping Lemma [Exam]

Are the following languages regular? Prove your claims!

a) $L_1 = \{0^a1^b0^c1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\}$

b) $L_2 = \{0^a1^b0^c1^d \mid a, b, c, d \geq 0 \text{ and if } a = 1 \text{ and } b = 2 \text{ then } c = d\}$
3 Transforming Automata [Exam]

Consider the DFA in Figure 2 over the alphabet $\Sigma = \{0, 1\}$ accepting the language $L$. Let $\Phi(L)$ be defined as

$$\Phi(L) := \{ w \in \Sigma^* \mid \exists x \in \Sigma^*, |x| = |w| \text{ and } wx \in L \}.$$ 

In other words, $\Phi(L)$ is the set of the front halves of all words in $L$.

![Figure 2: DFA B.](image)

a) Give a regular expression for the language $L$ accepted by the automaton $B$. If you like, you can do this by ripping out states as presented in the lecture (slide 1/84 ff.).

b) Construct a DFA which accepts a word $w$ if and only if $w \in \Phi(L)$. 
