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Distributed Computing

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Discrete Event Systems

Exercise Sheet 7

1 Night Watch

In order to improve their poor financial situation, Tobias and Klaus also work at nights. Their task is to guard a famous Swiss bank which, from an architectonic perspective, looks as follows:

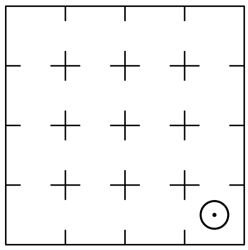

Figure 1: Offices of a Swiss bank.

Thus, there are 4x4 rooms, all connected by doors as indicated in the figure.

In a first scenario, Tobias and Klaus always stay together. They start in the room on the upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.

a) Compute the probability (in the steady state) that Tobias and Klaus are in the room where the thief enters the bank (indicated with $\odot$)!

b) Since Tobias and Klaus are very strong, they can easily catch a thief on their own. Thus, in a second scenario, they decide that it would be smarter to patrol individually (but they still start in the same room): After every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?

2 Hitting Time of a Simple Random Walk

a) Prove that in general the hitting time is not symmetric, i.e., that $h_{uv} \neq h_{vu}$.

b) Find a class of graphs in which $h_{uv} = h_{vu}$ holds for any two nodes u, v.

c) Describe a sufficient condition for $h_{uv} = h_{vu}$ in general graphs.

3 The Knight and the Bunny

Let $G = (V, E)$ be a connected undirected non–bipartite graph with $|V| = n$ and $|E| = m$. The Knight K and the Bunny B start on different nodes of G and both do a random walk, where they do one transition in one time unit. If they meet on the same node, then the Bunny slays the Knight. To solve the exercise, consider a Markov chain whose states are the ordered pair (k, b), where k is the position of the Knight and b is a position of the Bunny B.

Hint: A graph G is non-bipartite if and only if it contains an odd cycle.

a) What is the number of nodes and edges in the Markov chain?

b) Use slide "Cover Time von Random Walks" to bound h_{uv} for an edge from u to v in the Markov chain.

c) Show an upper bound of $12 \cdot n \cdot m^2$ for the expected time it takes the Bunny to kill the Knight by proving that there is always a path to a "meeting node" of length at most $3n$.