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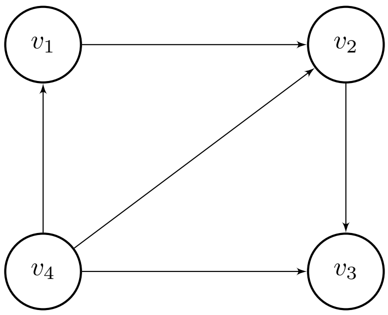
Discrete Event Systems

Exercise Sheet 8

1 PageRank

Usually, one would compute the PageRank of a network not by hand, but by running an algorithm on a computer – since the relevant instances are far too tedious to compute by hand. Still, we want you to get a feeling about the different versions of PageRank. Therefore we only consider a very small example with four nodes for this exercise:



a) Calculate the PageRank of the network according to the first version described on lecture slide 4/62 ("PageRank(1)").

b) Calculate the PageRank of the network according to the second version described on lecture slide 4/63 ("PageRank(2)").

c) Motivate the usage of the "Random Surfer" by calculating the PageRank of the network by using the iterative version described on lecture slide 4/64 ("PageRank(3)").
Notice: You should only need to iterate very few times.

2 Colour Blindness

Assume that the average rate of colour blindness is 2/100. (This is a very high rate, but it is supposed to make calculations easier. The actual rate is only about 1/100.000.)

a) Calculate the probability (exactly, i.e. not using an approximative distribution) that there is at most one colour blind person among a random sample of 100 persons. Calculate it as well using a Poisson-distribution.

b) What is the minimum size of a sample such that it contains at least one colour blind person with probability at least 90%? Now, you should assume that the number of colour blind people is Poisson-distributed with parameter $\lambda = np$ and thereby only obtain an approximative result.

The Poisson distribution

The Poisson distribution is a *discrete* probability distribution which is applied often to approximate the binomial distribution for large number n of repetitions and small success probability p of the underlying Bernoulli experiments. According to two frequently used rules of thumb, this approximation is good if $n \geq 20$ and $p \leq 0.05$, or if $n \geq 100$ and $np \leq 10$.
The Poisson distribution is often used to estimate the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event. The parameter $\lambda = np$ of the distribution is the expected number of occurrences in the interval.

$$\Pr[X = x] = \frac{\lambda^x}{x!}\, e^{-\lambda}$$