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Discrete Event Systems

Exercise Sheet 10

1 PhD-Scheduling [Exam]

In this assignment we wish to study the PHD-SCHEDULING-Problem, in which Professor Arno Nym wants to assign some of his more cumbersome tasks to his m PhD students. When Prof. Nym enters his office in the morning there is a pile of tasks on his desk, and he is only able to see the topmost task at a time. He wants do decide *online* to which of his m PhD students he assigns the topmost task before taking a look at the next task. Prof. Nym does not know in advance when the last task is reached, because there might be more tasks hidden beneath the topmost one. Every task needs to be assigned to exactly one PhD student, and every PhD student is able to solve any task. Of course, Prof. Nym would like all tasks to be finished as fast as possible.

A task is defined by its *effort* $e_i \in \mathbb{R}^+$, independent of the PhD student who processes the task, and different tasks may require different effort. The *load* L_j of a PhD student j is defined by the sum of the efforts of all tasks assigned to j. He thus needs time L_j to finish all assigned tasks. Prof. Nym's goal is that all tasks are finished in as little time as possible, which means that the maximum load of any PhD student is to be minimized. For a sequence of tasks $\sigma = (e_1, e_2, \ldots)$, the cost of an assignment $\mathcal{A}$ of tasks to students is thus defined by

$$\text{cost}(\mathcal{A}(\sigma)) := \max_{j=1,\ldots,m} L_j(\mathcal{A}(\sigma)).$$

In the following we analyze the algorithm SMALLLOAD that assigns the topmost task to a student whose load is minimal with respect to the previous assignments.

a) Assume that Prof. Nym has only two PhD students, i.e., $m = 2$.

(i) Describe the execution of SMALLLOAD for the following sequence of tasks σ.

$$\sigma = (2,\ 5,\ 4,\ 3,\ 7)$$

What would be an optimal solution for this sequence? How large is the competitive ratio of SMALLLOAD with respect to σ?

(ii) The cost of a solution found by SMALLLOAD can be worse than that of an optimal solution. Construct a sequence of tasks σ' for which the costs of SMALLLOAD are as high as possible compared to the costs of an optimal solution.

(iii) Is the algorithm SMALLLOAD c-competitive for some constant c? If so, give the smallest possible such c. Prove your claim!

(iv) Is there an efficient way to compute an optimal *offline* solution? Explain your answer.

b) Now, analyze the case in which the number of PhD students m is arbitrary. What is the smallest competitive ratio of SMALLLOAD now? Prove your claim!

2 Queuing Networks

Customers of the Internet Service Provider *RedWindow* who have problems with their Internet access, can call a hot-line. There, a customer must first talk to a dispatcher. The dispatcher is very moody and with probability p_d, he kicks people out of the line. However, with probability $1 - p_d$, a customer is connected to a technician. The technician can solve the problem with probability p_t. However, if he cannot solve it, he claims that the problem is the fault of the monopolistic modem producer *Beep*. Thus, with probability $1 - p_t$, the customer has to call *Beep*. Unfortunately, the agent at Beep can solve the problem only with probability p_b. With probability $1 - p_b$, the customer is told that RedWindow is the source of the problem, and hence the customer is connected back to the dispatcher of RedWindow. And so on and so forth...

In the following, we assume that a customer calling RedWindow for the second time experiences exactly the same success probabilities as in the first round. Let now the arrival times of the *direct* (i.e., not reconnected) calls to RedWindow be Poisson distributed with parameter λ. Moreover, assume that the technician of RedWindow and the agent of Beep do not get additional (direct) calls. The service times of the dispatcher, the technician and the agent are exponentially distributed with parameter μ_d (dispatcher), μ_t (technician) and μ_b (Beep agent). If the dispatcher, the technician or the agent are occupied, the customer is put into the waiting line of the corresponding person.

a) Model the situation using the techniques from the lecture.

b) Describe the arrival rate of the phone calls at the technician of RedWindow as a function of p_d, p_t, p_b and λ!

c) How long is a customer in the waiting queue of the technician after he has been forwarded from the dispatcher until he is eventually served (on average)?

d) Now assume that $p_d = 1/6$, $p_t = 1/5$, $p_b = 1/4$, and $\lambda = 5$ per hour. Moreover, let $\mu_d = 20$ per hour, $\mu_t = 10$ per hour, and $\mu_b = 10$ per hour. Compute the expected number of customers in the system (of both RedWindow and Beep together)! What is the expected time a customer is in the system?

3 A Night at the DISCO [Exam]

An entertainment entrepreneur asks you to help him dimension rooms for his DISCO. The establishment consists of a dance floor, a bar, and the restrooms. The arrivals of visitors to the DISCO can be modeled as a Poisson process with rate λ. Visitors enter the DISCO at the dance floor. The sojourn time there is exponentially distributed with parameter μ_d. With probability p_v the visitor dislikes what the DJ plays and leaves the DISCO; with probability p_b dancing makes her thirsty and she goes to the bar.

At the bar, visitors order drinks. The service rate at the bar (ordering with the bar team, mixing, and drinking) is μ_b drinks per minute. Afterwards, with probability p_d the visitor goes to the dance floor. On the other hand, with probability p_r, before going back to the dance floor, the visitor has to go to the restrooms, where she spends an amount of time exponentially distributed with parameter μ_r.

a) Model the DISCO as a queuing network.

b) State the arrival rate for the dance floor as a function of λ, p_b, p_r, p_v and p_d.

c) Data shows that roughly 90 people visit the restrooms per hour, and that the average time spent there is 5 minutes. How many toilets should be installed to ensure that the queue does not grow indefinitely? (Assume that a toilet can be used by only one guest at a time.)

d) The business consultant "Toilets-R-Us" claims that the expected time it takes for the first guest to use the restroom after opening the DISCO can be calculated simply as $\lambda + \mu_d + \mu_b$. This is of course incorrect. Find the mistake!