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Computer Engineering and Networks Laboratory
Computer Engineering Group (TEC)

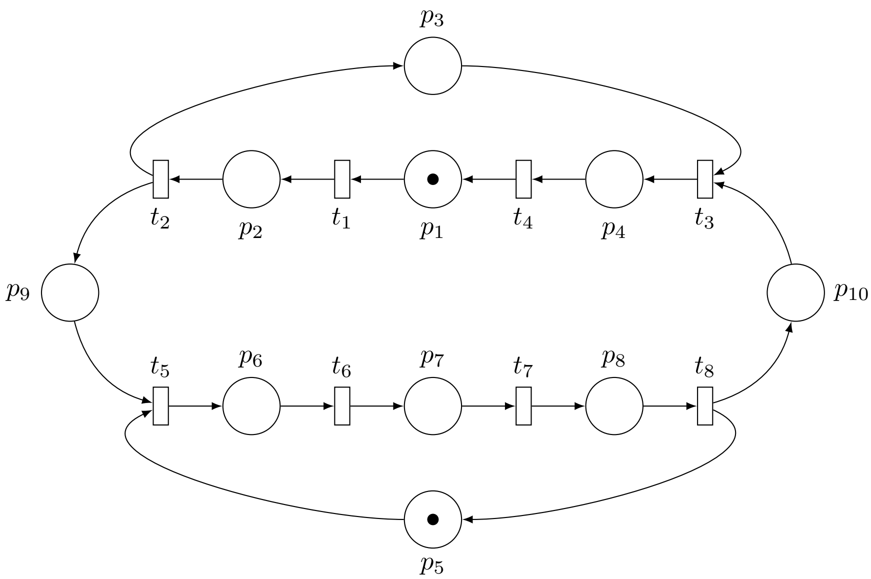
HS 2015 Prof. L. Thiele / R. Jacob

Discrete Event Systems

Exercise Sheet 13

1 Structural Properties of Petri Nets and Token Game

Given is the following Petri net N_1:



a) What are the Pre and Post sets of transitions t_5 and t_8 and of place p_3?

b) Which transitions are enabled after t_1 and t_2 fired?

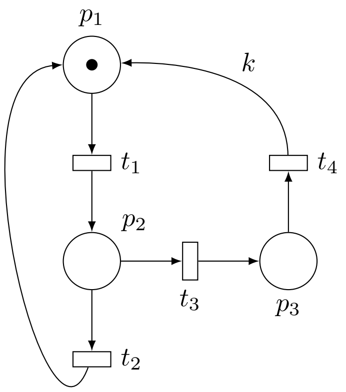
c) What is the total number of tokens in N_1 before and after t_2 fired?

d) Play the token game for N_1 and construct the reachability graph.

Hint: You may denote the states in such a way that the index indicates the places that hold a token in this state, for example $\vec{s}_0 = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0) \triangleq s_{1,5}$.

2 Basic Properties of Petri Nets

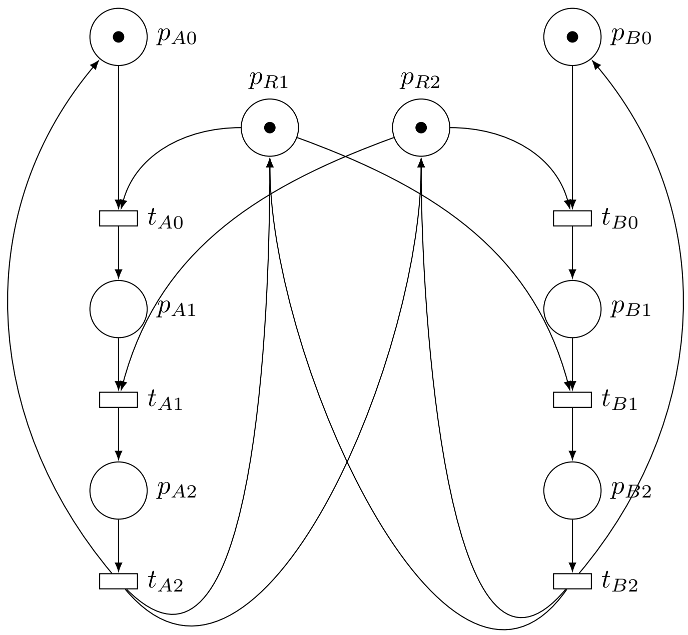
Given is the following Petri net N_2:



Explain the terms *boundedness* and *deadlock-freeness* using this example, i.e. for which values of $k \in \mathbb{N}$ is the Petri net N_2 bounded/unbounded and not deadlock-free?

3 Identifying a deadlock

The following Petri net N_3 describes two linear processes ($P_{A0/A1/A2}$ and $P_{B0/B1/B2}$) sharing resources R_1 and R_2.



In the following, use $M = (P_{A0}, P_{A1}, P_{A2}, P_{R1}, P_{R2}, P_{B0}, P_{B1}, P_{B2})$ as marking vector and $T = (t_{A0}, t_{A1}, t_{A2}, t_{B0}, t_{B1}, t_{B2})$ as firing vector.

a) Determine the reachability graph of this net for the given initial marking. Explicit one or several firing sequences leading to a blocking marking (i.e., to a deadlock). What is this blocking marking?

b) Write down the upstream (W^-) and downstream (W^+) incidence matrices and deduce the incidence matrix A. Use it to compute the marking obtained in the deadlock state (i.e., by firing the blocking sequence) from the previous question.

c) Using the upstream incidence matrix W^-, how can you **prove** that this previous state is a deadlock?

d) Suggest a modification to this Petri net which allows the two linear processes P_A and P_B to run as intended in the first place.

4 From mutual exclusion to starvation

Your task is to model a system as a Petri net in which two processes want to access a common exclusive resource is a similar fashion as in Exercise 3. This means that the two processes have to exclude each other mutually from the concurrent access to the resource (e.g. a critical program section). More precisely:

1. A process executes its program.
2. In order to enter the critical section, a given mutex variable must be 0.
3. If this is the case, the process sets the mutex to 1 and executes its critical section.
4. When done, it resets the mutex to 0 and enters an uncritical section.
5. Then the procedure starts all over again.

a) Propose a Petri net representing the desired behavior.
Hint: use 5 places and 4 transitions.

b) In this setting, it may happen that a process starves the other. That means one process always uses the resource and the other never enters the critical section. Correct this in such a way that each process cannot get the resource more than twice in a row. This may yield that only one process can start running from the initial marking.

5 Reachability Analysis for Petri Nets

In the lecture we presented an algorithm to perform a reachability analysis on Petri nets.

a) Why is it not possible with a reachability algorithm to determine *in general*, whether a given state in a Petri net is reachable or not?

b) Consider the Petri net N_2 from exercise 2. Is the state $s = (p_1 = 101, p_2 = 99, p_3 = 4)$ reachable from the initial state $s_0 = (1, 0, 0)$ if $k = 2$? Prove your answer.

Hint: Start with the necessary condition presented in the lecture for the reachability of a state in a weighted Petri net, then eventually explain whether or not the marking is reachable.

6 Coverability tree and graph

Given is the following Petri net N_6, compute its coverability tree and coverability graph. Deduce which are the unbounded places of this net given the initial marking.