Discrete Event Systems
Solution to Exercise Sheet 14

This last exercise session tries to put everything together somehow. You will discover a modeling tool for time Petri nets, which is paramount in the evaluation of a system design. You will mainly use the "stepper simulator", which randomly plays the token game. In the last part of the exercise, you will use the embedded model-checker to tune some design parameters.

1 Warming up: Calculating with Petri nets

In this exercise you are supposed to model a function $f_i(x, y)$ using a Petri net. That is, the Petri net must contain two places $P_x$ and $P_y$ that hold $x$ and $y$ tokens respectively in the beginning. Additionally, the net must contain one place $P_z$ which holds $f_i(x, y)$ tokens when the net is dead. The Petri nets are supposed to work for arbitrary numbers of tokens in $P_x$ and $P_y$.

- a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$
- b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$
- c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$

Hint: You may have to use inhibitor arcs for c). An inhibitor arc between a place and a transition prevents the transition from firing as long as there is at least one token in the place.

There are multiple options here. I just present one set of solutions. These are captures of TINA models of these nets. The models are available with the solution.

2 Simulate your Petri nets with TINA

In this exercise, you will learn how to use the modeling tool TINA (TIme petri Net Analyzer). The objective is to implement your solutions from Exercise 1 and simulate the nets.

- a) On my webpage (http://people.ee.ethz.ch/~jacobr/teaching.html) you will find a simple tutorial for basic uses of the TINA software. Complete the reading of Section 1 before moving on.

- b) To practice, create new net files and implement your solutions from Exercise 1. Using the stepper, try them out for different values of $x$ and $y$. 
3 Queue sizing and overflow management of BOLT

BOLT is an ultra-low power processor interconnect that decouples arbitrary application and communication processors with respect to time, power and clock domains. BOLT supports asynchronous message passing with predictable timing characteristics, and therefore making it possible for the system designer to construct highly-customized platforms that are easier to design, implement, debug, and maintain. You can find more information on the webpage: http://www.bolt.ethz.ch/

In this exercise, we look down to the practical problem of queue sizing and overflow management, using BOLT architecture as a case study.

Assume on one end BOLT is connected to a sensor, which delivers data at a given frequency and writes into BOLT queue. At the other end, a communication processor is in charge to read these messages from BOLT and to send them through a wireless sensor network. The problem is that the network is not always available (other processes also need the bandwidth) and the communication processors have limited memory to store messages prior sending them.

The task is to evaluate the design parameters (queue size, resource management scheme, ...) such that overflows in the BOLT queue (which would result in packet losses) are avoided.

3.1 The nice and easy deterministic world

a) Download the initial Petri net (Ex3.1) from my webpage: http://people.ee.ethz.ch/~jacobr/teaching.html. Open it in nd.

b) This initial model is untimed. Extend it with delays on transitions such that the net follows the earliest firing rule and knowing that:

- Task of reading and writing to BOLT as well as sending a message to the network take one time unit each.
- The sensor produces one message every 5 time units.
- At steady state, the communication processor can use the network for 10% of the time.

It should be straightforward to obtain the following model

![Petri net diagram]

c) Play the token game using the stepper. Is the net bounded? Why is it not surprising?

This net is obviously not bounded because it generates one packet every 5 time units on one end, and disposes of one every 10 time units only on the other end. Hence, packets pile up in the middle.

d) Assume BOLT has now a maximal capacity of two massages. How would modify the net to implement this?

*Hint:* The easiest way is to add a capacity to the BOLT place...

There are two easy ways of doing this. Either directly add a capacity to the BOLT place or use a weighted inhibitor arc from BOLT over the writing transition.
On the physical system, BOLT cannot prevent a write operation if its queue is full. Hence, the previous solution to avoid overflows is not satisfying, as it yields that the write operation is forbidden if there is no more memory available in BOLT, so the systems is actually waiting for the resource to be available. In our case, we would like to be sure instead that we have enough network resource available such that there is no overflow in BOLT, without controlling the write operations.

As a result, we need our Petri net to model the (possible) overflow of BOLT in order to test our design. Among other options, we choose to do that by adding a sink transition downstream the BOLT place, as shown thereafter.

If it happens that 3 tokens are in the BOLT place, the overflow transition is enabled and can fire. This is used as a flag to detect an overflow of BOLT.

e) In our case, we want to avoid overflows by increasing the amount of network resource available. More precisely, we want to double it. This can be done in several ways:

- Assume you get the network 20% of the time.
- Assume there are two networks, that you get 10% of the time each. Further assume the communication processor can read one message per network (e.g., if there are two networks, you should have 2 tokens in place p4). You can either model explicitly two independent networks, or set a second token in the network available place.

These three options are illustrated below

Try out these options. Why solution c) does not solve our problem? Which of solution a) or b) seems the more flexible to you?
The option c) does not work because of the mechanism of delay reset in time Petri nets. In our case, when there are two tokens in network busy and t4 fires, this creates an evolution of the enabling of t4, hence the delay is recomputed (as explained in the lecture). Since the delay cannot be anything else than 9 time units, it starts over from there, regardless how long the second token has already spent in the place. This implies that the resource has in fact a cycle time of 9 time units, and not 5 as we wanted.

Furthermore, we will emphasis with the next question that solution b) is more flexible, as you can send 2 messages anytime during a 10 time units window, while for a), you must wait 5 time units between two messages.

f) Assume now the incoming packets arrive in burst of 5 messages every 10 time units. Consider again solutions of type a) and b) with increased bandwidth:

- One network that you get 50% of the time.
- Five networks that you get 10% of the time each.

Model the bursts and try out these options. Which solution seems the more flexible to you now?

As mentioned before, option b) is more flexible, it can absorb bursts that a) cannot. The reason is that the communication processor cannot read out fast enough the messages from

![Diagram](image-url)
BOLT, one every two time units at the most. If you allow multiple reads (e.g., 5 tokens in p4), it makes options a) and b) equivalent.

As you can see, even in this very basic scenario, a correct analysis of the system design is not so easy. Coming up with correct design parameters is hard. In practice, it is almost impossible for real-life systems. This is why test and verification methods have been developed and increasingly used over the past decades.

3.2 Realism yields non-determinism

Let us now consider a slightly more realistic model.

a) Download the new Petri net structure (Ex3.2) from my webpage: http://people.ee.ethz.ch/~jacobr/teaching.html. Open it in nd.

For each task, there is no precise execution time anymore but a time interval instead. This represents the Best- and Worst-Case Execution Time (BCET/WCET). The resource management mechanism and the communication processor are also different.

- The processor can read and store several messages from BOLT before sending them.
- When the processor accesses the network to send messages, it will keep on sending (i.e., no releasing the resource) until it has no more message left in memory.
- It is uncertain when the network will be available again.

b) Complete the reading of Section 2 of the TINA tutorial.

The question is to determine how much memory we need in the communication processor to ensure that BOLT will never overflow, no matter what happens. To do that, we will use the LTL model-checker embedded within TINA. LTL is a different logic than CTL. For our simple purposes here, we can say the difference is that the $E$ quantifier doesn’t exist and the $A$ quantifier is always implied. Furthermore, $G$ writes $\Box$, $F$ writes $\Diamond$, and $X$ writes $\Diamond$. This is summarized in the table below:

<table>
<thead>
<tr>
<th>CTL</th>
<th>AF a</th>
<th>AG a</th>
<th>EF a</th>
<th>AX a</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTL</td>
<td>$\Diamond$ a</td>
<td>$\Box$ a</td>
<td>$\neg$ AG $\neg$ a</td>
<td>$\Diamond$ a</td>
</tr>
</tbody>
</table>

in TINA $<>$ a ; $[]$ a ; $\neg$ $[]$ a $\Diamond$ a ; $\Diamond$ a ; ...

c) What does the LTL property $\Diamond t5$ mean in natural language? Does our model verify this property? Are we happy about this?

$\Diamond t5 \equiv$ “Whatever happens, eventually transition $t5$ will fire.”

The initial model verifies this property. This is of course the opposite of what we want!! It means there will always be an overflow of BOLT at some point, even in the best-case scenario...

d) What is the LTL property our system must verify in order to guarantee that BOLT will never overflow?

We want that ”Whatever happens, there is no overflow of BOLT”. This can be verified with the LTL query ”$\Box \neg t5$” evaluating to TRUE. This is equivalent to the CTL ”AG $\neg(t5)$”.

e) What is the smallest capacity for the place p4 that makes this true.

For this model, the communication processor needs to be able to read at least 27 messages to absorb the worst case of messages arrival in BOLT and of the network availability.