Discrete Event Systems
Exercise Sheet 10

1 Comparison of Finite Automata

Here are two simple finite automata:

For each, we have a one bit encoding for the states (\(x_A\) and \(x_B\)), one binary output (\(y_A\) and \(y_B\)), and one common binary input (\(u\)). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

a) Express the characteristic function of the transition relation for both automaton, \(\psi_r(x, x', u)\).

b) Express the joint transition function, \(\psi_f\).
   \[ \psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u)) \]

   Reminder: \(\psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))\).

c) Express the characteristic function of the reachable states, \(\psi_X(x_A, x_B)\).

d) Express the characteristic function of the reachable output, \(\psi_Y(y_A, y_B)\).

e) Are the two automata equivalent? **Hint:** Evaluate, for example, \(\psi_Y(0, 1)\).
2 Temporal Logic

a) We consider the following automaton. The property $a$ is true on the colored states (0 and 3).

![Diagram of automaton]

For each of the following CTL formula, list all the states for which it holds true.

(i) $EF \ a$
(ii) $EG \ a$
(iii) $EX \ AX \ a$
(iv) $EF (\ a \ AND \ EX \ NOT(a))$

b) Given the transition function $\psi_f(q, q')$ and the characteristic function $\psi_Z(q)$ for a set $Z$, write a small pseudo-code which returns the characteristic function of $\psi_{AFZ}(q)$. It can be expressed as symbolic boolean functions, like $x_A x_A' x_B x_B' + x_A x_A' x_B x_B'$. 

**Hint:** To do this, simply use the classic boolean operators $\text{AND}$, $\text{OR}$, $\text{NOT}$ and $\neq$. You can also use the operator $\text{PRE}(Q, f)$, which returns the predecessor of the set $Q$ by the transition function $f$. That is,

$$\text{PRE}(Q, f) = \{q' : \exists q, \psi_f(q', q) \cdot \psi_Q(q) = 1\}$$

**Hint:** It can be useful to reformulate $\text{AFZ}$ as another CTL formula.