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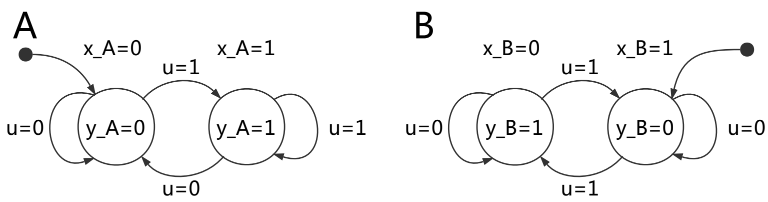
# Discrete Event Systems

## Exercise Sheet 10

## 1 Comparison of Finite Automata

Here are two simple finite automata:




For each, we have a one bit encoding for the states ($x_A$ and $x_B$), one binary output ($y_A$ and $y_B$), and one common binary input ($u$). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

**a)** Express the characteristic function of the transition relation for both automaton, $\psi_r(x, x', u)$.

**b)** Express the joint transition function, $\psi_f$.
**Reminder:** $\psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))$.

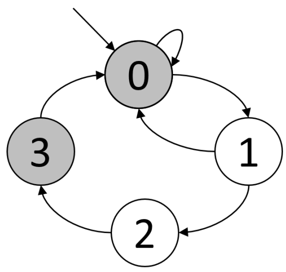
**c)** Express the characteristic function of the reachable states, $\psi_X(x_A, x_B)$.

**d)** Express the characteristic function of the reachable output, $\psi_Y(y_A, y_B)$.

**e)** Are the two automata equivalent? **Hint:** Evaluate, for example, $\psi_Y(0, 1)$.

## 2 Temporal Logic

**a)** We consider the following automaton. The property $a$ is true on the colored states (0 and 3).




For each of the following CTL formula, list all the states for which it holds true.

(i) EF $a$

(ii) EG $a$

(iii) EX AX $a$

(iv) EF ( $a$ AND EX NOT($a$) )

**b)** Given the transition function $\psi_f(q, q')$ and the characteristic function $\psi_Z(q)$ for a set $Z$, write a small pseudo-code which returns the characteristic function of $\psi_{\text{AF } Z}(q)$. It can be expressed as symbolic boolean functions, like $\overline{x_A} x'_A \overline{x_B} x'_B + \overline{x_A} x'_A x_B x'_B$.
**Hint:** To do this, simply use the classic boolean operators AND, OR, NOT and $! =$. You can also use the operator PRE$(Q, f)$, which returns the predecessor of the set $Q$ by the transition function $f$. That is,

$$\texttt{PRE}(Q, f) = \{q' : \exists q,\ \psi_f(q', q) \cdot \psi_Q(q) = 1\}$$

**Hint:** It can be useful to reformulate AF$Z$ as another CTL formula.