Discrete Event Systems
Exercise Sheet 11

1 Comparison of Finite Automata

Here are two simple finite automata:

For each, we have a one bit encoding for the states ($x_A$ and $x_B$), one binary output ($y_A$ and $y_B$), and one common binary input ($u$). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

a) Express the characteristic function of the transition relation for both automaton, $\psi_r(x, x', u)$.

b) Express the joint transition function, $\psi_f$.
   Reminder: $\psi_f(x_A, x_A', x_B, x_B') = (\exists u : \psi_A(x_A, x_A', u) \cdot \psi_B(x_B, x_B', u))$.

c) Express the characteristic function of the reachable states, $\psi_X(x_A, x_B)$.

d) Express the characteristic function of the reachable output, $\psi_Y(y_A, y_B)$.

e) Are the two automata equivalent? Hint: Evaluate, for example, $\psi_Y(0, 1)$. 
2 Temporal Logic

a) We consider the following automaton. The property $a$ is true on the colored states (0 and 3).

For each of the following CTL formula, list all the states for which it holds true.

(i) EF $a$
(ii) EG $a$
(iii) EX AX $a$
(iv) EF ($a$ AND EX NOT($a$))

b) Given the transition function $\psi_f(q, q')$ and the characteristic function $\psi_Z(q)$ for a set $Z$, write a small pseudo-code which returns the characteristic function of $\psi_{AFZ}(q)$. It can be expressed as symbolic boolean functions, like $x_A x'_A x_B x'_B + x_A x'_A x_B x'_B$.

**Hint:** To do this, simply use the classic boolean operators AND, OR, NOT and $\neq$. You can also use the operator \texttt{PRE}(Q, f), which returns the predecessor of the set $Q$ by the transition function $f$. That is,

$$\text{PRE}(Q, f) = \{ q' : \exists q, \psi_f(q', q) \cdot \psi_Q(q) = 1 \}$$

**Hint:** It can be useful to reformulate $AFZ$ as another CTL formula.