

Discrete Event Systems

Exercise Sheet 4

1 Pumping Lemma Revisited

Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

2 Context Free or Not?

For the following languages, determine whether they are context free or not. Prove your claims!

- a) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |z|, |x| = |y|\}$
- b) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |y|, |x| = |z|\}$

3 Pushdown Automata: Reloaded

Consider the following context-free grammar G with non-terminals S and A , start symbol S , and terminals “(”, “)”, and “0”:

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

- a) What are the eight shortest words produced by G ?
- b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.
- c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

Remark: a) and b) are taken from Exercise Sheet 3.

4 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L .

- a) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{“}u \text{ is a palindrome”}\}$
- b) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{“}u \text{ is no palindrome”}\}$