Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Exercise Sheet 8

1 Queuing Networks

Customers of the Internet Service Provider *RedWindow* who have problems with their Internet access, can call a hot-line. There, a customer must first talk to a dispatcher. The dispatcher is very moody and with probability p_d, he kicks people out of the line. However, with probability $1 - p_d$, a customer is connected to a technician. The technician can solve the problem with probability p_t. However, if he cannot solve it, he claims that the problem is the fault of the monopolistic modem producer *Beep*. Thus, with probability $1 - p_t$, the customer has to call *Beep*. Unfortunately, the agent at Beep can solve the problem only with probability p_b. With probability $1 - p_b$, the customer is told that RedWindow is the source of the problem, and hence the customer is connected back to the dispatcher of RedWindow. And so on and so forth...

In the following, we assume that a customer calling RedWindow for the second time experiences exactly the same success probabilities as in the first round. Let now the arrival times of the *direct* (i.e., not reconnected) calls to RedWindow be Poisson distributed with parameter λ. For simplicity, you can assume that arrivals in the network can always be modeled as Poisson distributions. Note that this is not exact for networks with loops.

Moreover, assume that the technician of RedWindow and the agent of Beep do not get additional (direct) calls. The service times of the dispatcher, the technician and the agent are exponentially distributed with means $1/\mu_d$ (dispatcher), $1/\mu_t$ (technician) and $1/\mu_b$ (Beep agent). If the dispatcher, the technician or the agent are occupied, the customer is put into the waiting line of the corresponding person.

a) Model the situation using the techniques from the lecture.

b) Assuming stability at each node, describe the arrival rate of the phone calls at the technician of RedWindow as a function of p_d, p_t, p_b and λ!

c) Once again, assume stability at each node. How long is a customer in the waiting queue of the technician after he has been forwarded from the dispatcher until he is eventually served (on average)?

d) Now assume that $p_d = 1/6$, $p_t = 1/5$, $p_b = 1/4$, and $\lambda = 5$ per hour. Moreover, let $\mu_d = 20$ per hour, $\mu_t = 10$ per hour, and $\mu_b = 10$ per hour. Compute the expected number of customers in the system (of both RedWindow and Beep together)! What is the expected time a customer is in the system?

2 A Night at the DISCO

An entertainment entrepreneur asks you to help him dimension rooms for his DISCO. The establishment consists of a dance floor, a bar, and the restrooms. The arrivals of visitors to the DISCO can be modeled as a Poisson process with rate λ. Visitors enter the DISCO at the dance floor. The sojourn time for a person there is exponentially distributed with mean $1/\mu_d$. With probability $p_v = 1 - p_b$ the visitor dislikes what the DJ plays and leaves the DISCO; with probability p_b dancing makes her thirsty and she goes to the bar.

At the bar, each visitor orders one drink. The service rate at the bar (ordering with the bar team, mixing, and drinking) is μ_b drinks per minute (of course, exponentially distributed). Afterwards, with probability p_d the visitor goes back to the dance floor. On the other hand, with probability $p_r = 1 - p_d$, before going back to the dance floor, the visitor has to go to the restrooms, where she spends an amount of time exponentially distributed with mean $1/\mu_r$.

- **a)** Model the DISCO as a queuing network.
- **b)** Then, assuming stability, state the arrival rate for the dance floor as a function of λ, p_b, p_r, p_v and p_d. For simplicity, we assume that the *total* arrival rate for the dance floor can be modeled as a Poisson distribution. Note that this is not exact for networks with loops.
- **c)** Data shows that roughly 90 people visit the restrooms per hour, and that the average time spent there is 5 minutes. How many toilets should be installed to ensure that the queue does not grow indefinitely? (Assume that a toilet can be used by only one guest at a time.)
- **d)** The business consultant "Toilets-R-Us" claims that the expected time it takes for the first guest to use the restroom after opening the DISCO can be calculated simply as $\lambda + \mu_d + \mu_b$. This is of course incorrect. Find the mistake!