Discrete Event Systems

Exercise session #3

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1. Pumping Lemma

Is following language regular?

$$L = \{0^a 1^b 0^c 1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\}$$

1. Pumping Lemma

Assume for contradiction that L is regular, p is the pumping length.

Let $w = 0110^p 1^p$, $w \in L$ and $|w| > p$.

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* If y consists of only 0s from the second block,

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We therefore consider the various cases.

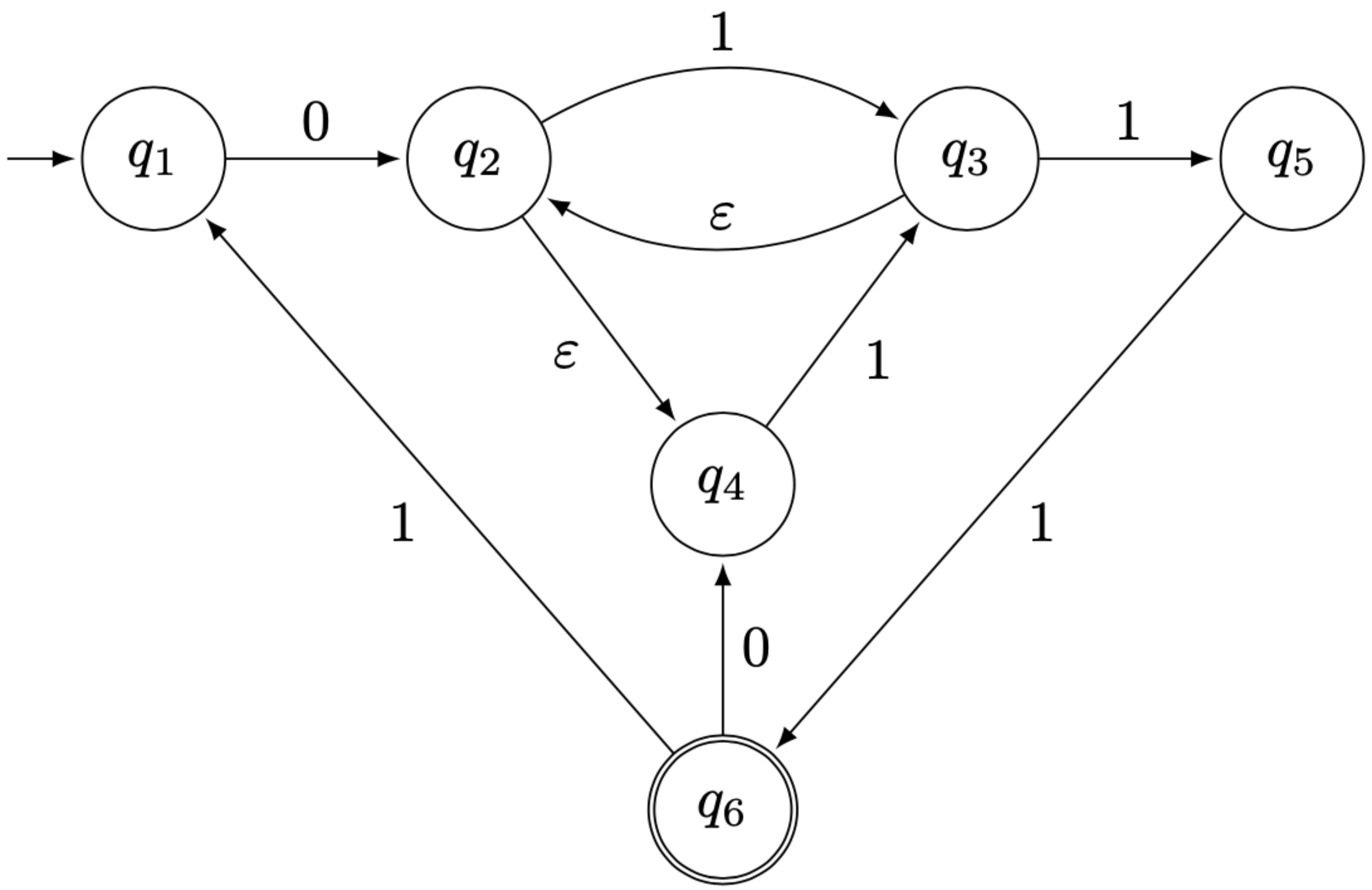
* If y starts anywhere within the first three symbols (i.e. 011) of w, deleting y creates a word with an illegal prefix (e.g. $1\,0^p1^p$ for $y = 01$).
* If y consists of only 0s from the second block, the word $w' = xy^2z$ has more 0s than 1s in the last $|w'| - 3$ symbols and hence $c \mathrel{!}= d$.

Note that y cannot contain 1s from the second block because of the requirement $|xy| \leq p$.

Therefore, L cannot be regular and we have a contradiction.

2. Deterministic Finite Automata [Exam]

Transform the NFA into an equivalent DFA, while assuming Σ = {0,1}.
(Hint: Only construct states which are necessary!)



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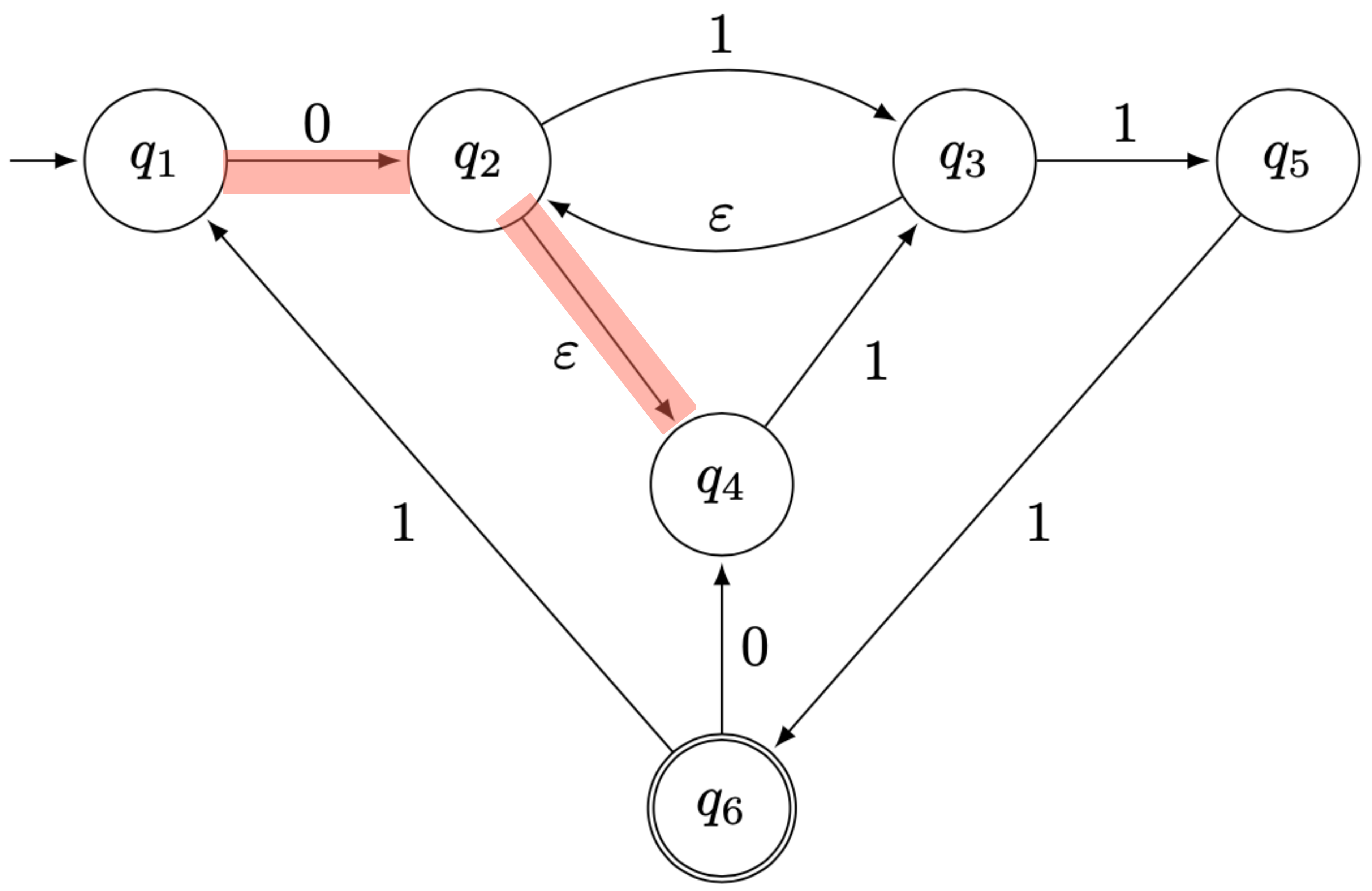
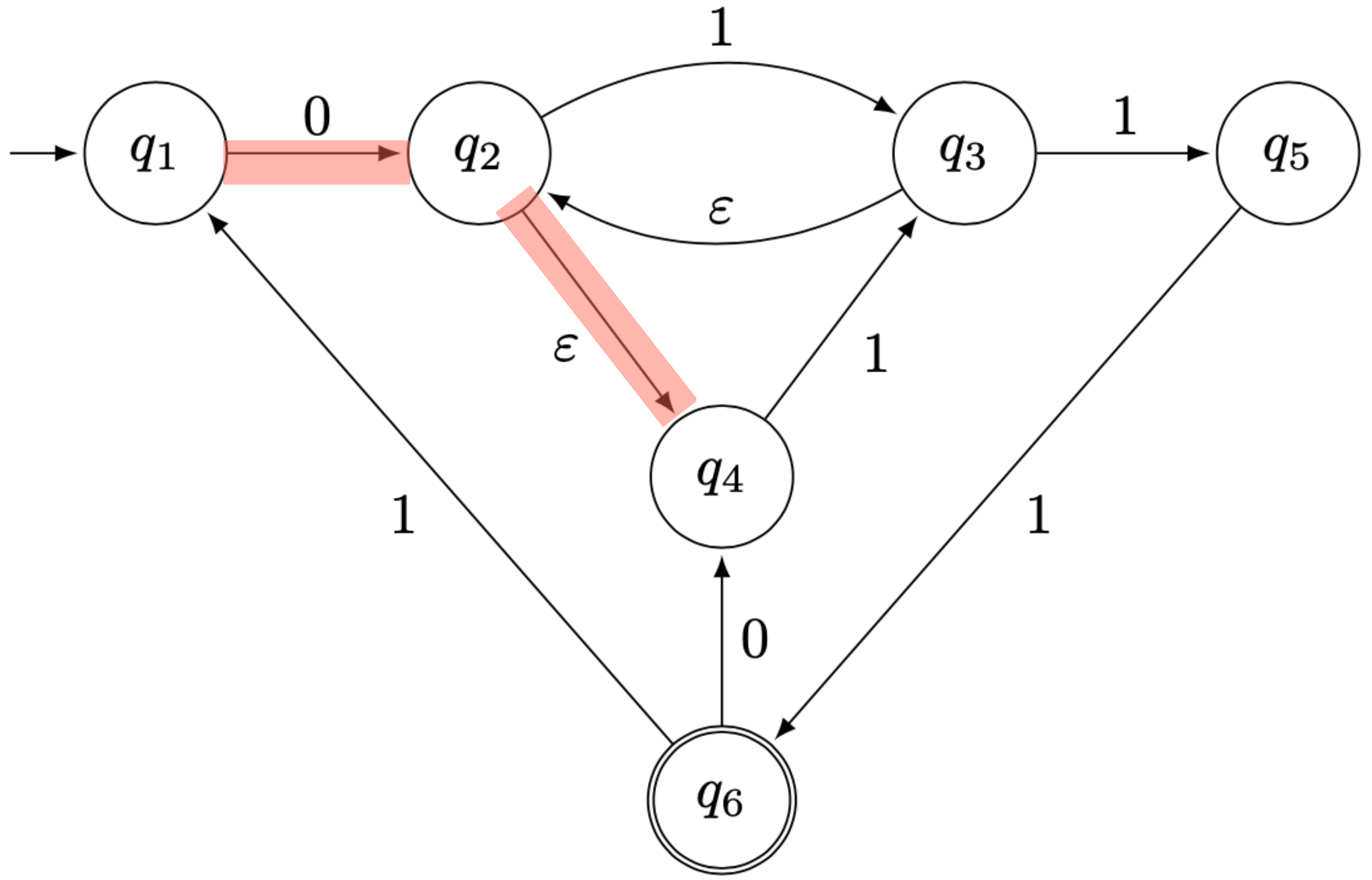


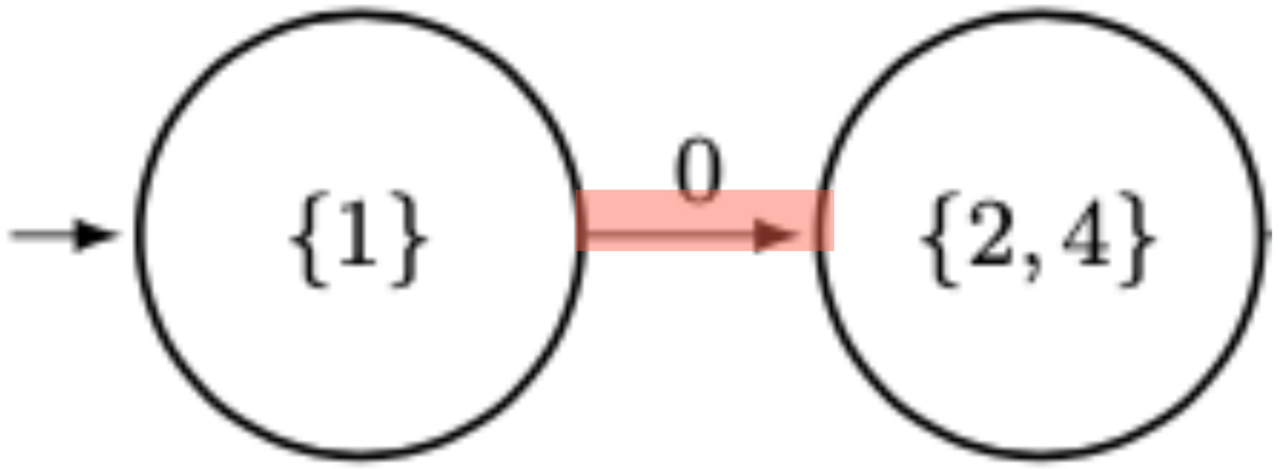
NFA

2 Deterministic Finite Automata [Exam]



NFA

DFA



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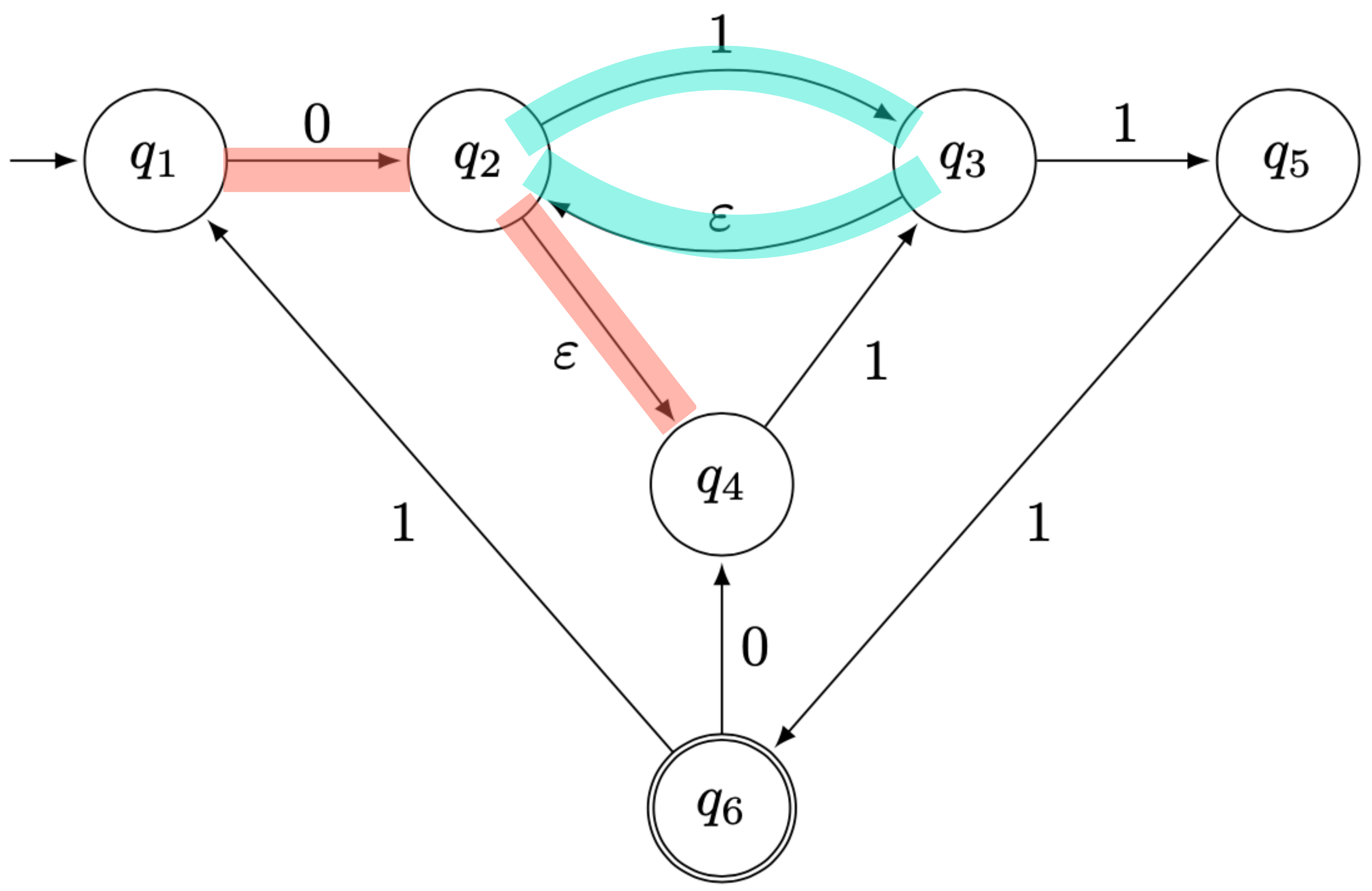

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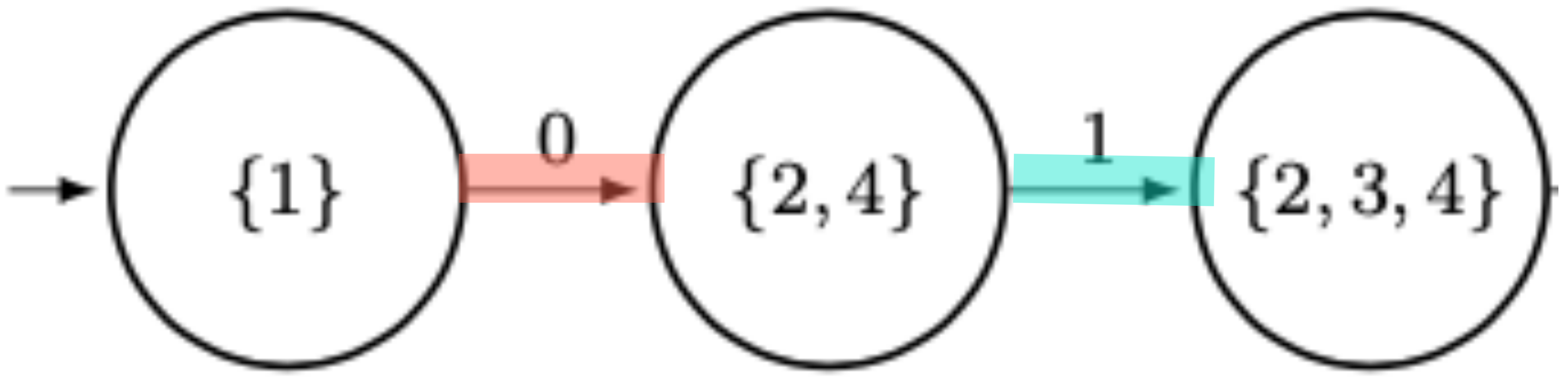

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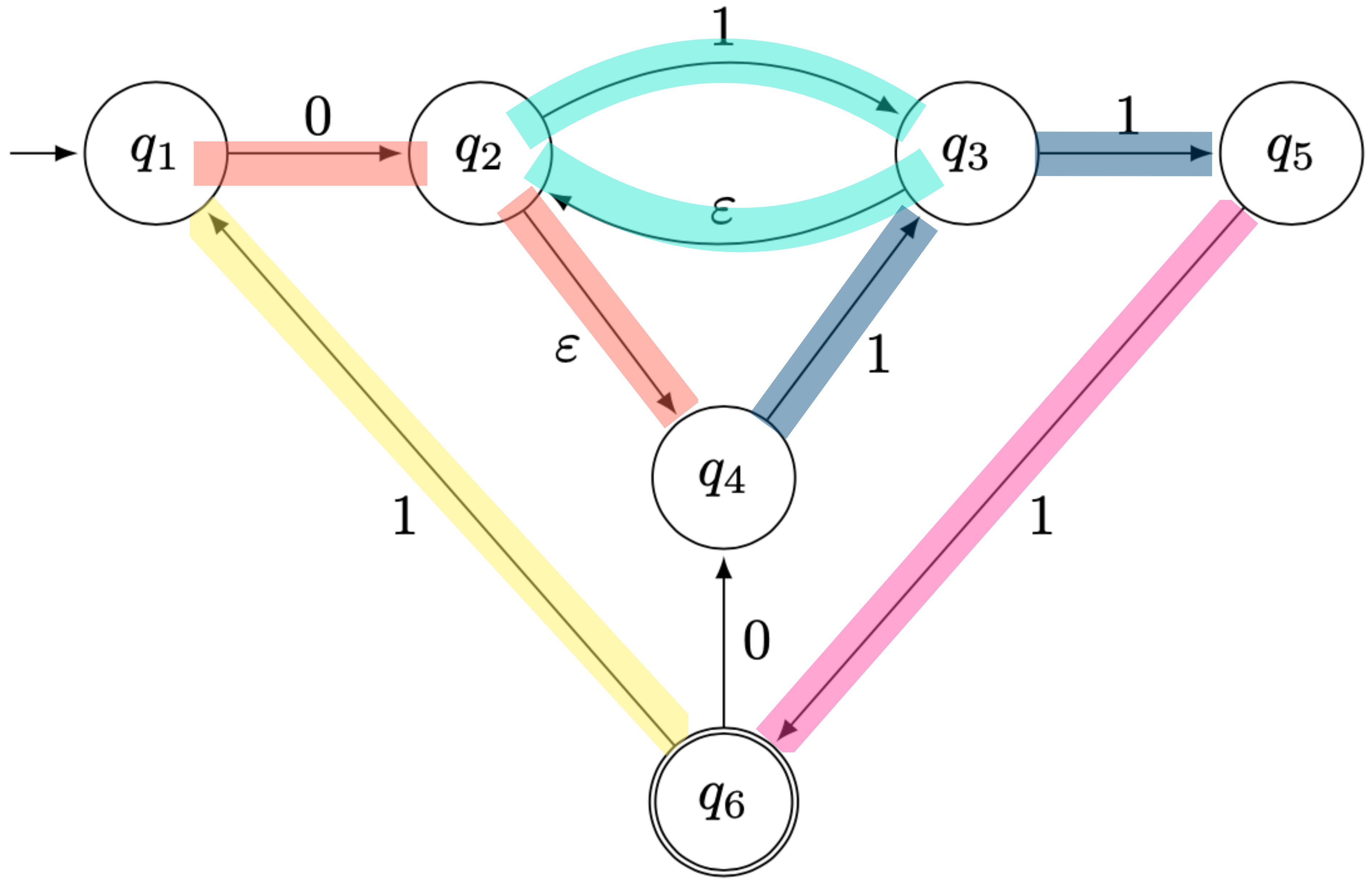

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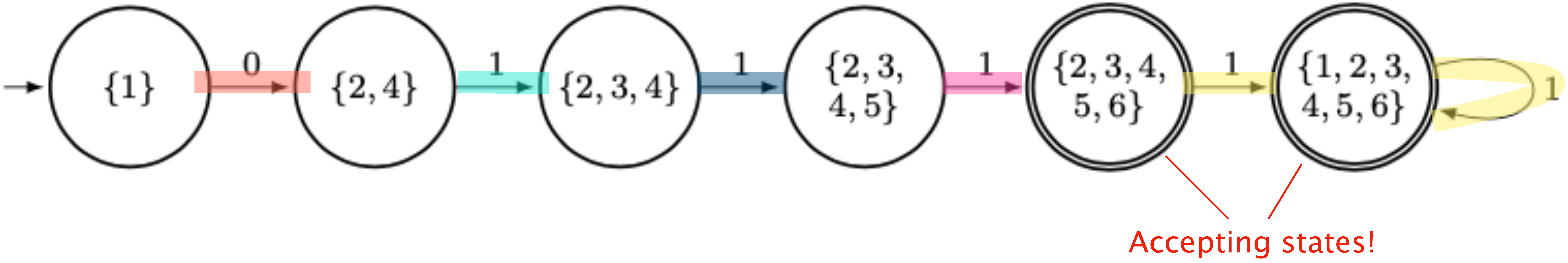

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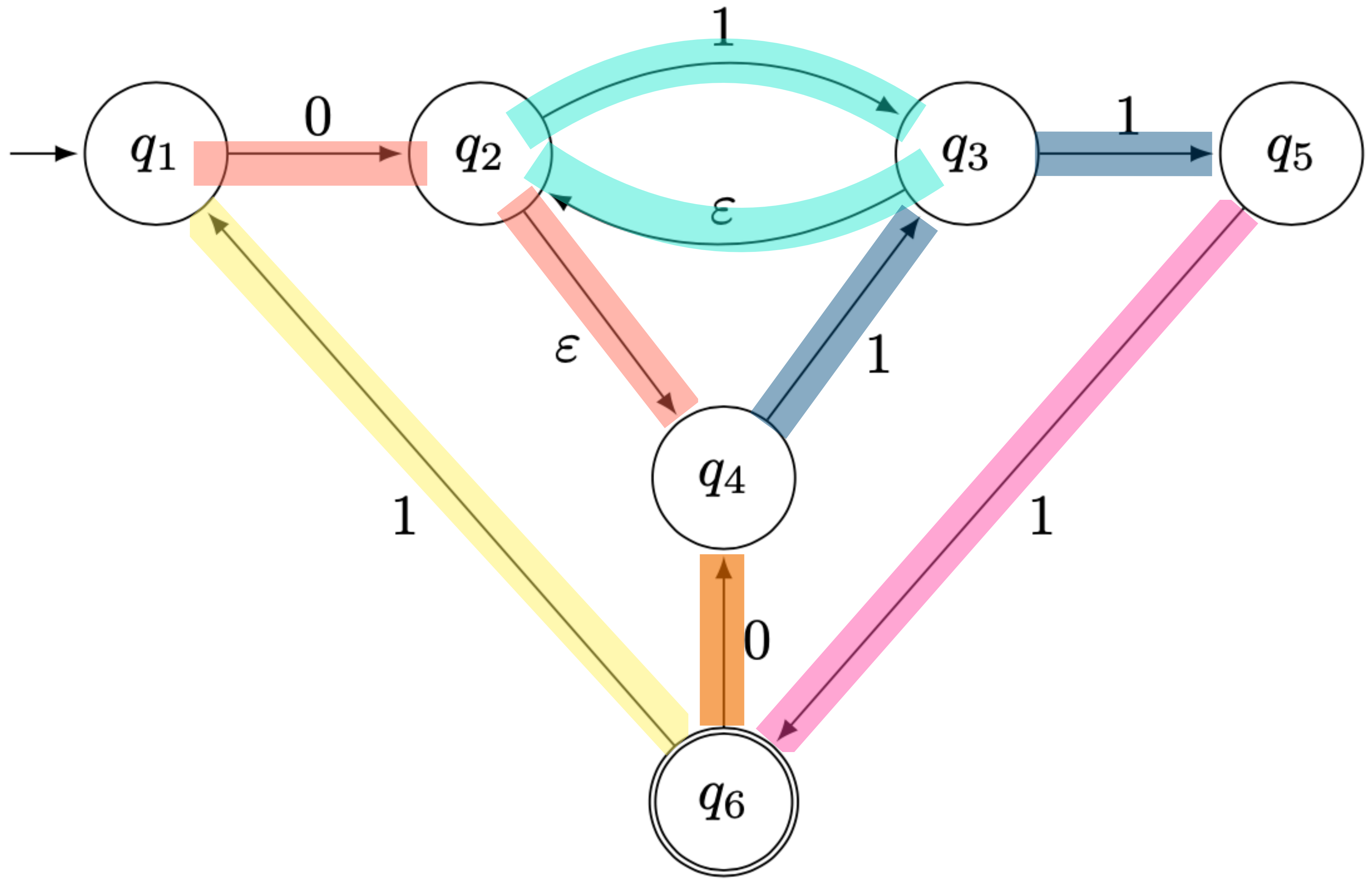
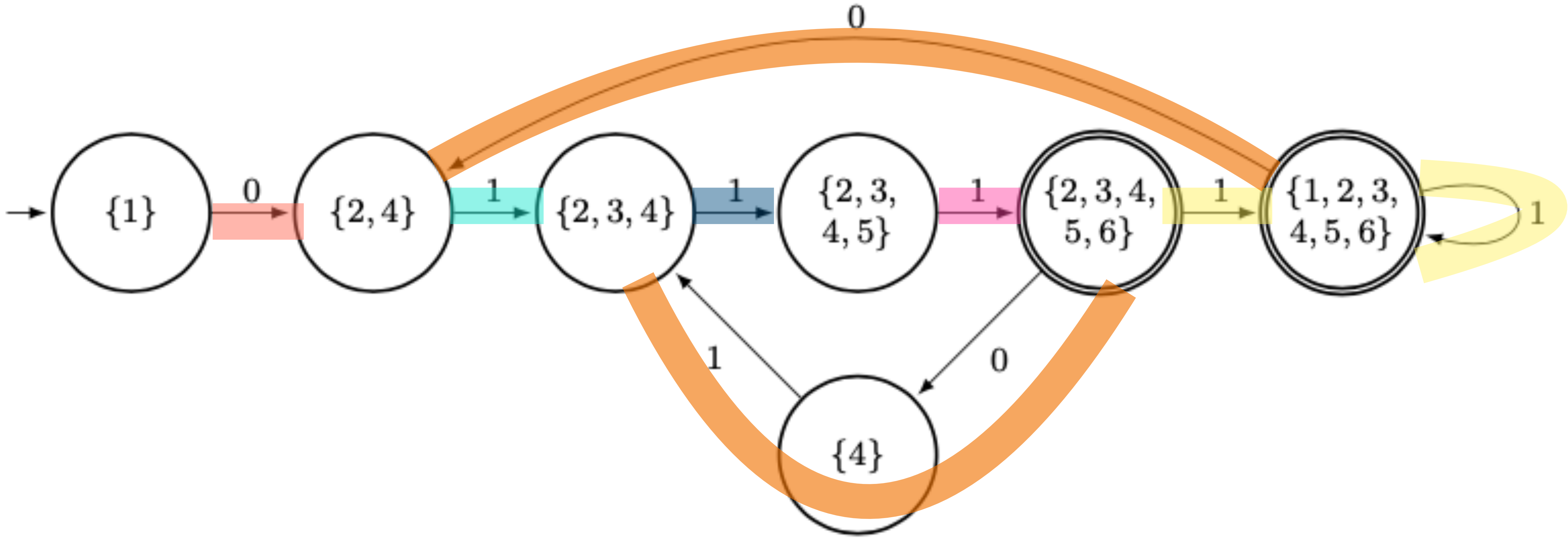


DFA



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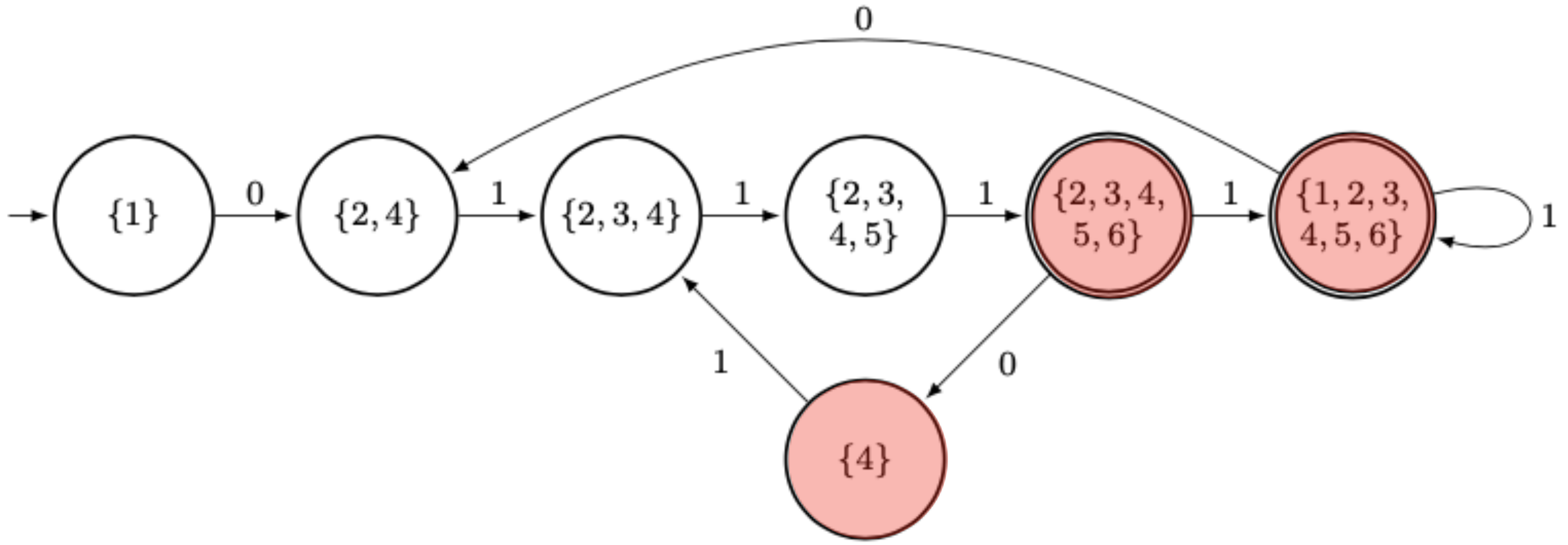

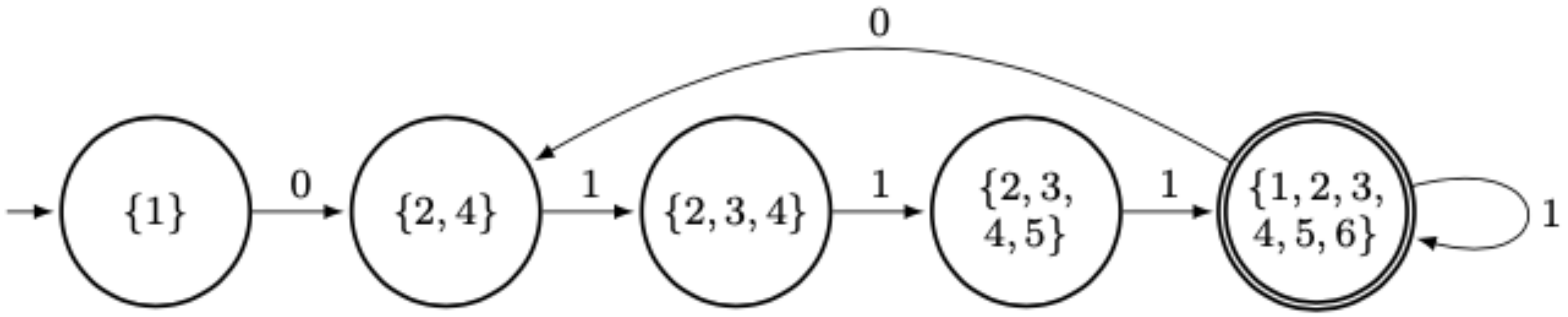

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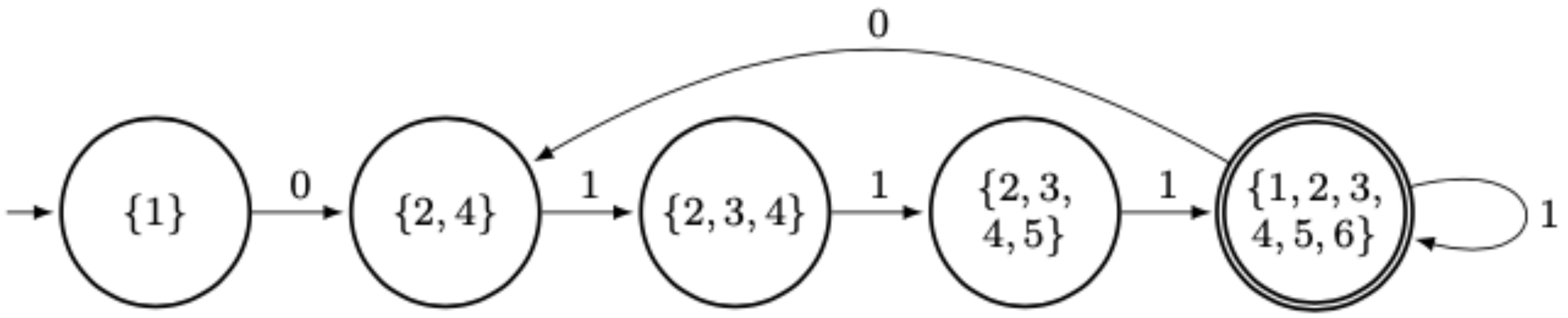
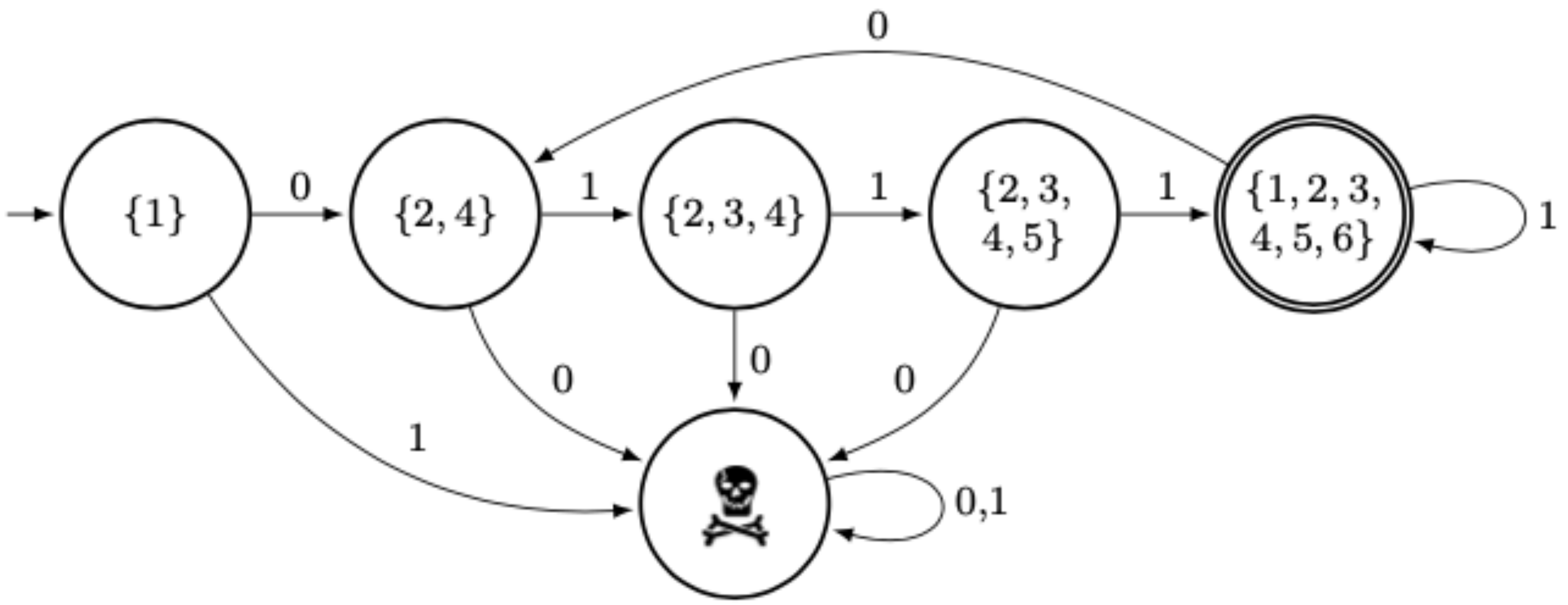
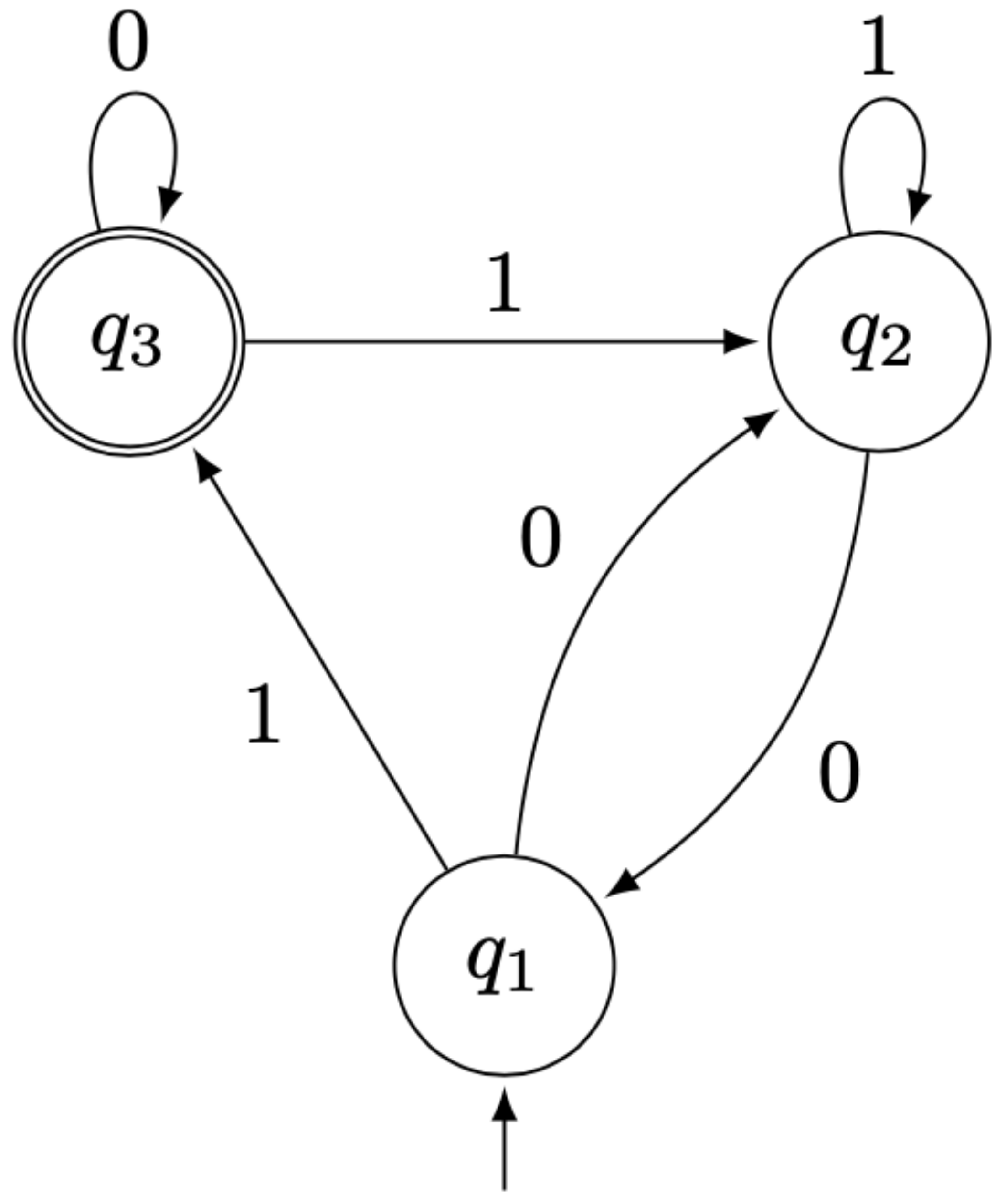




3. Transforming Automata [Exam]

Consider the DFA over the alphabet Σ = {0, 1}. Give a regular expression for the language L accepted by the automaton below. If you like, you can do this by ripping out states as presented in the lecture.



Hint: remove q2, q1, q3

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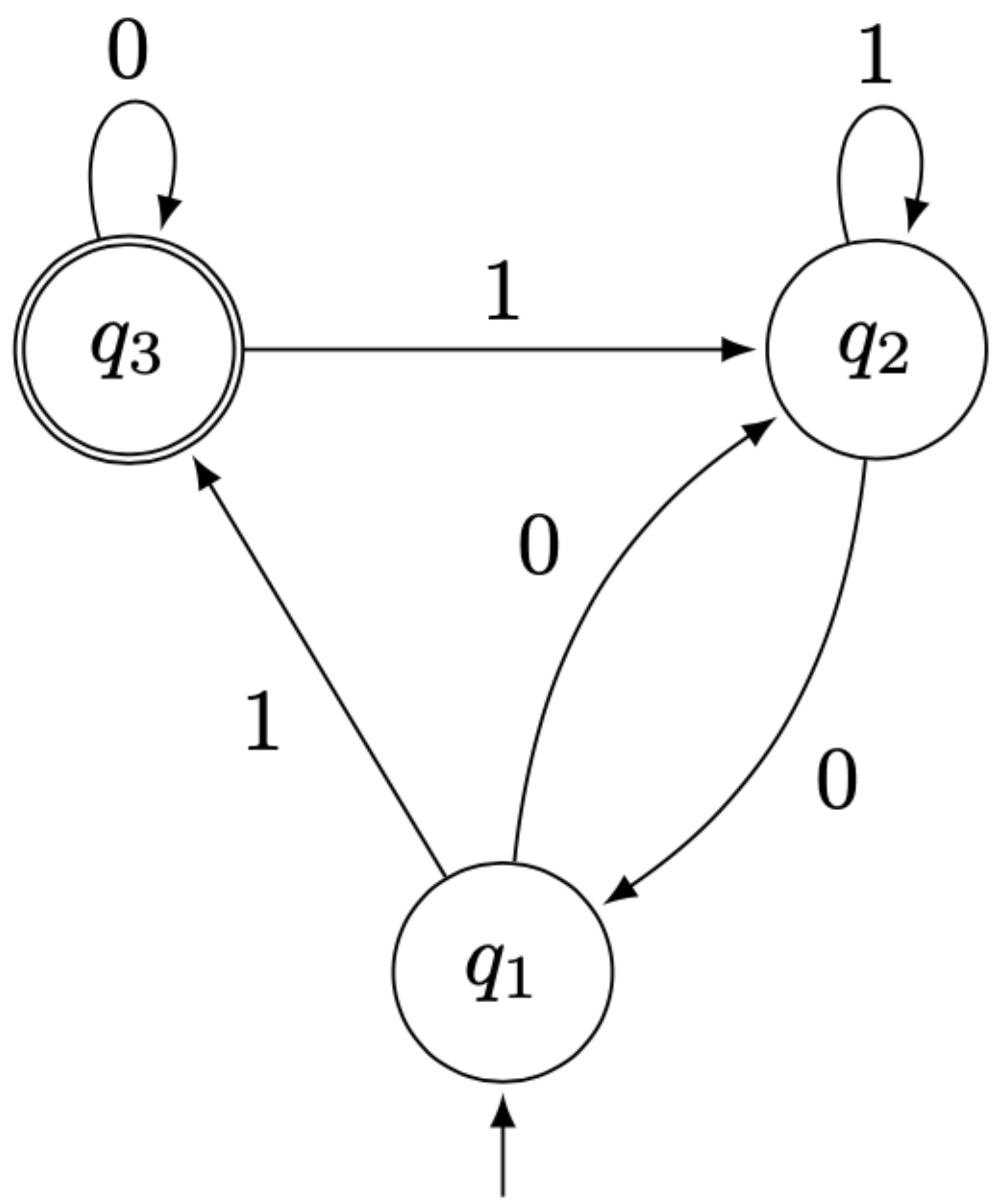
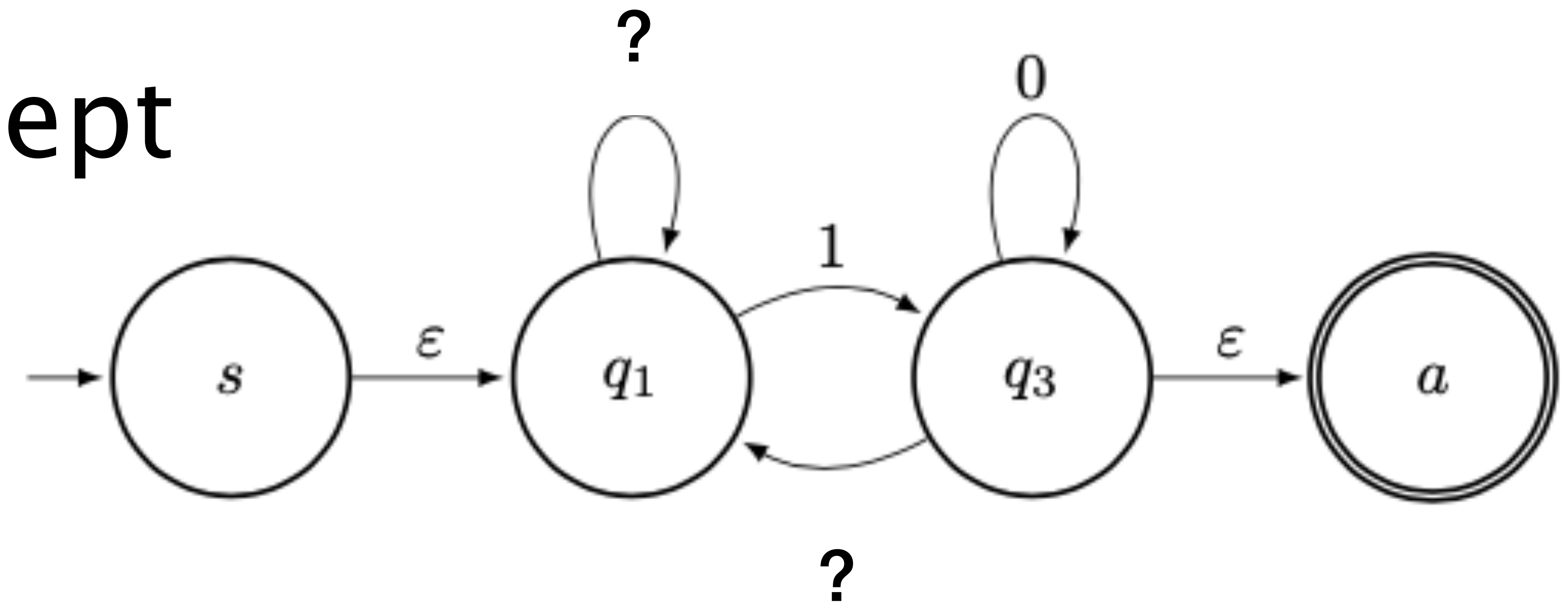


Add start and accept

Ripe out q2



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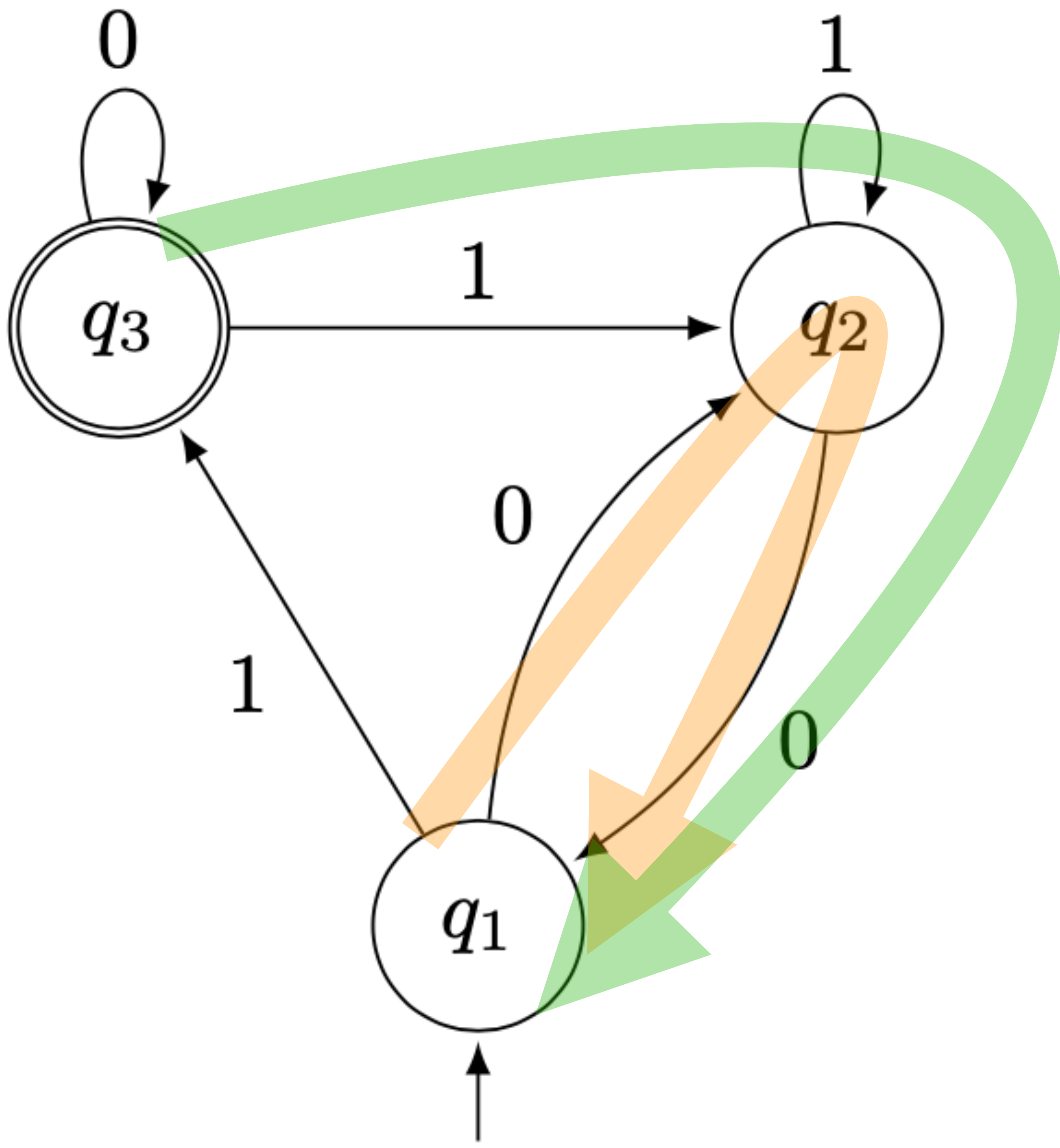

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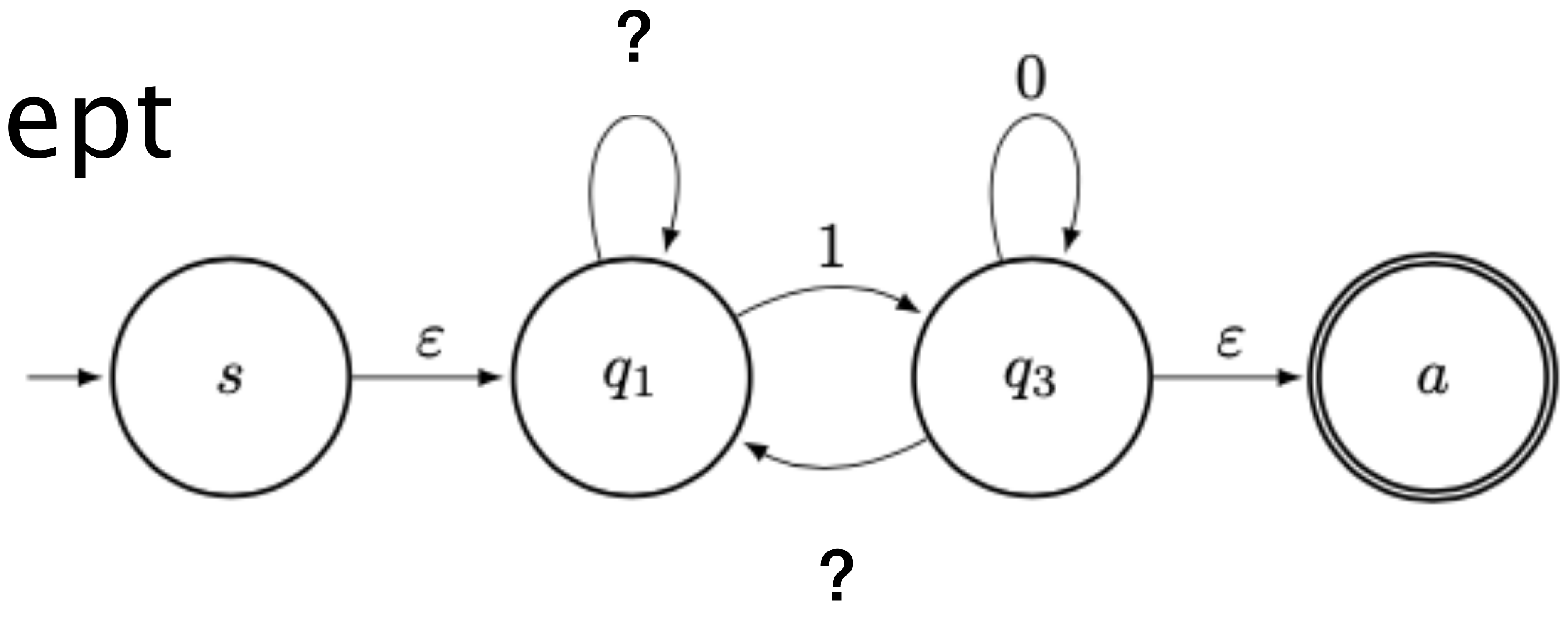

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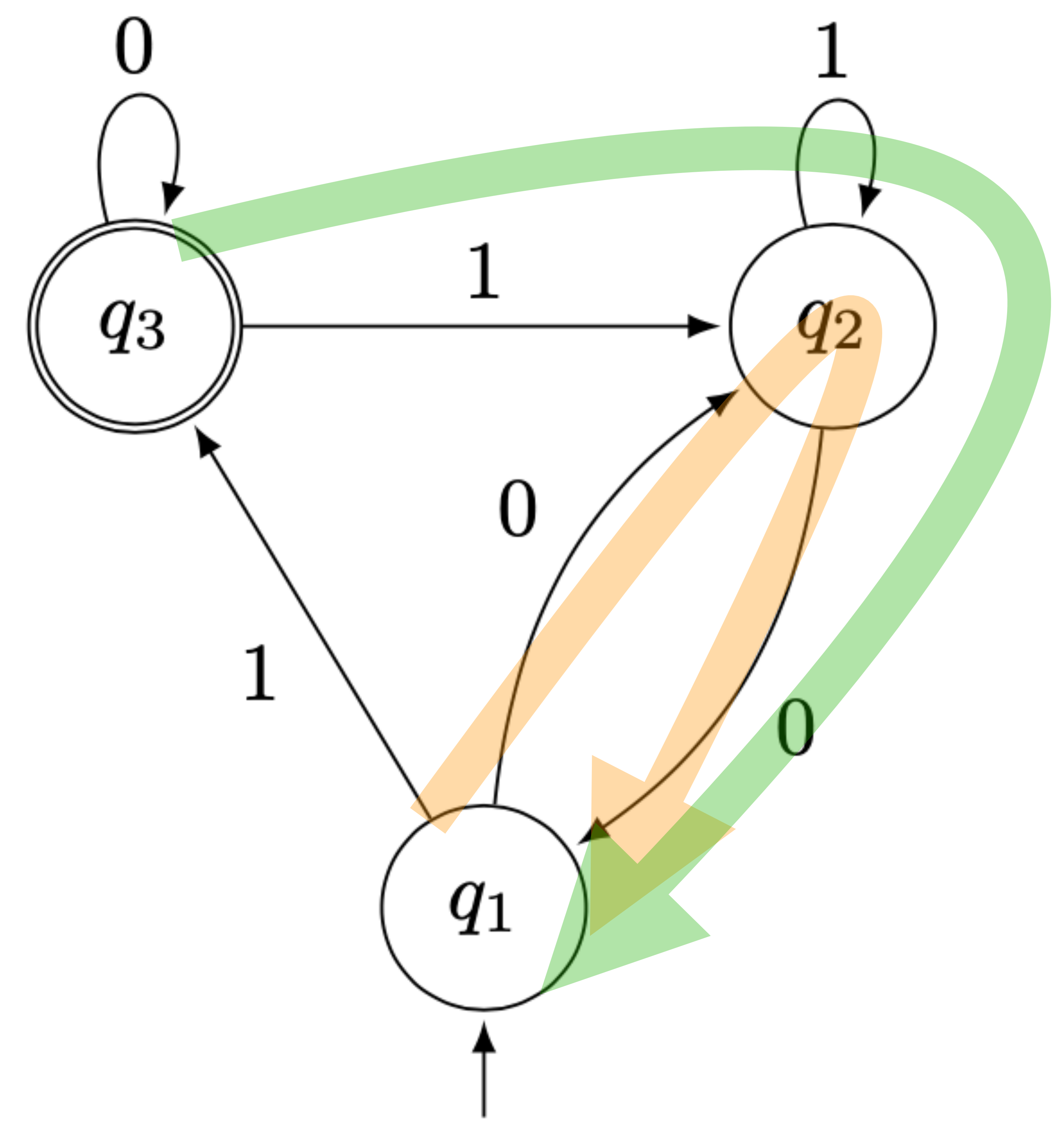

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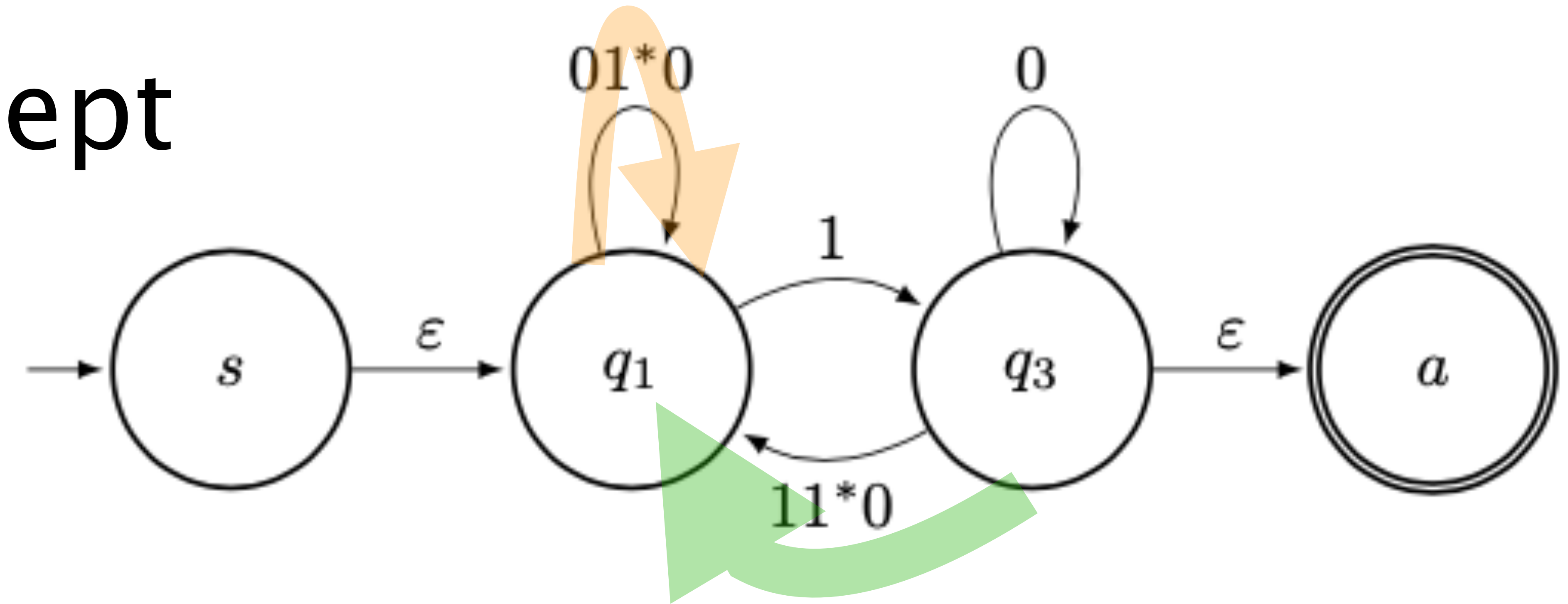

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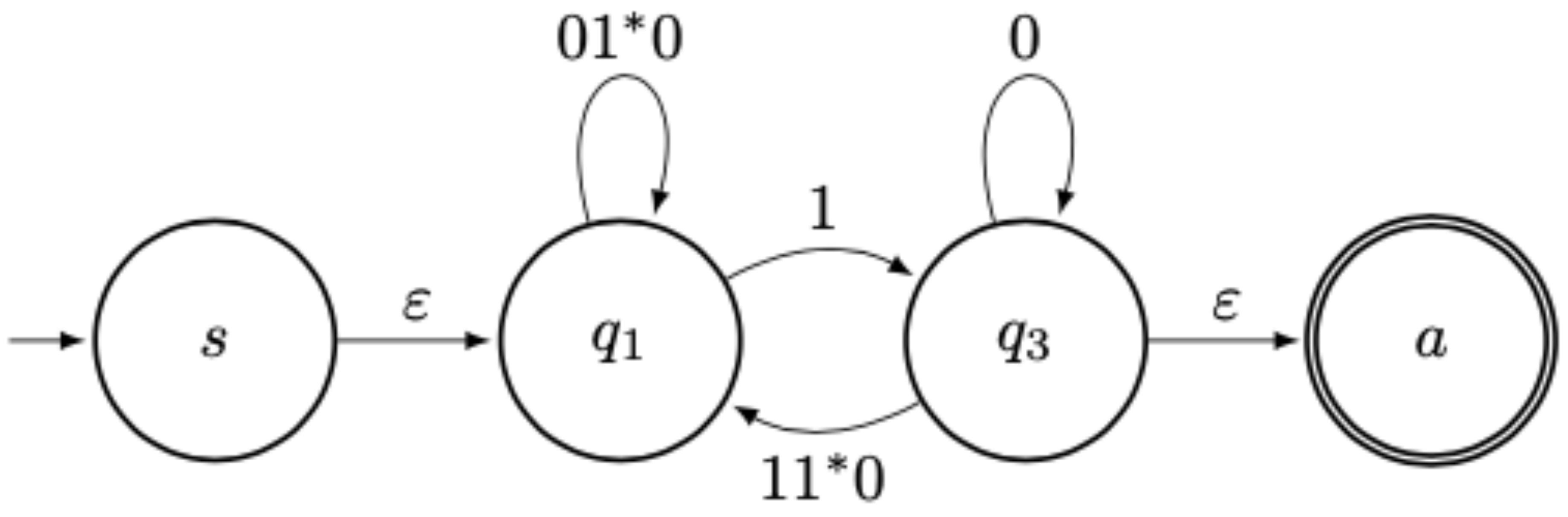
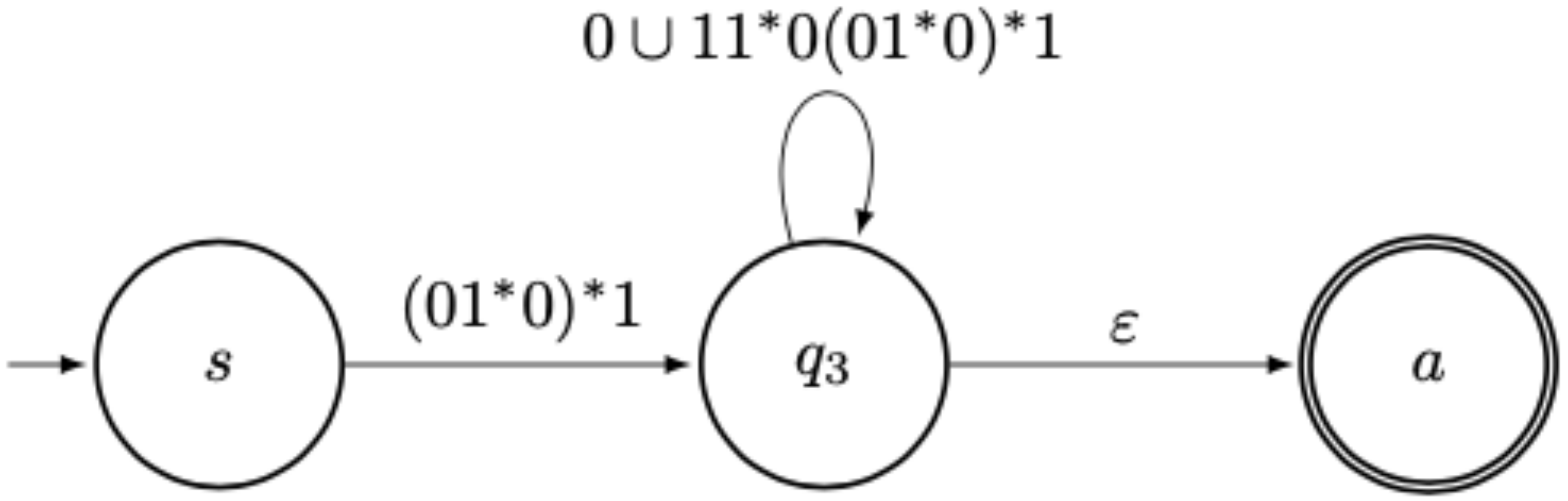


Ripe out q1



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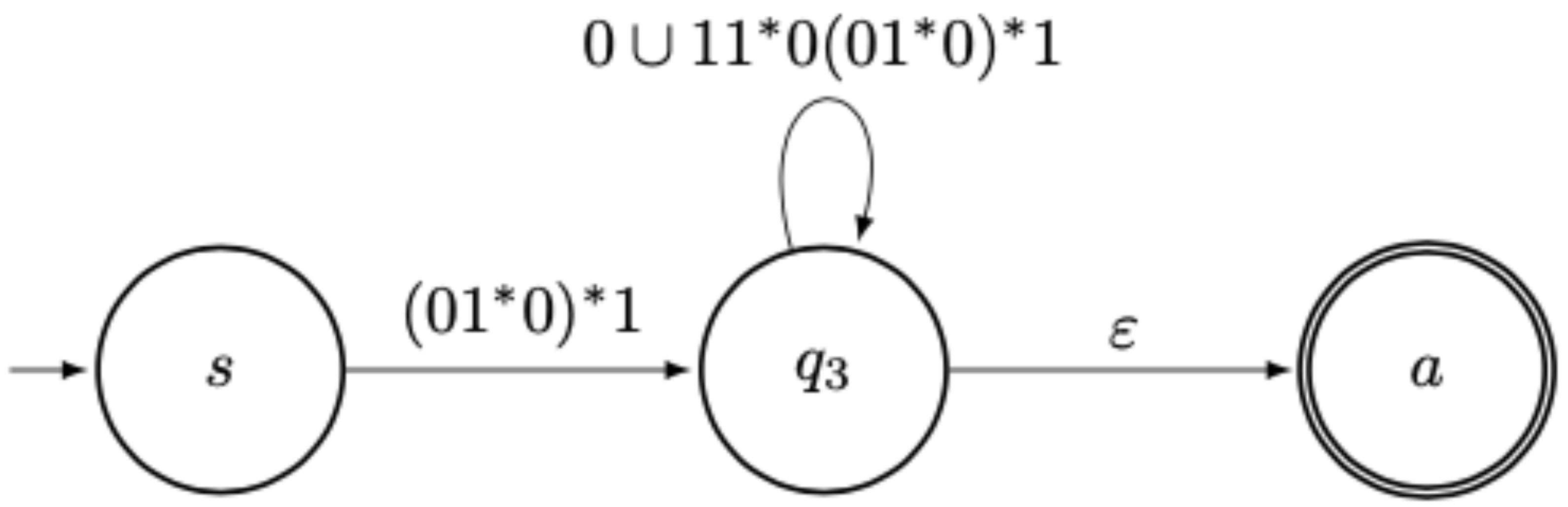

Ripe out q3

$$(01^*0)^*1(0 \cup 11^*0(01^*0)^*1)^*$$

4. Pumping Lemma

a) $L = 1^n 0 2^n$ $>=0$

Is L regular?

Assume L is regular.

We take $w = 1^p 0\, 2^p \in L$,

$w = xyz$ with $|xy| \leq p$ and $|y| \geq 1$, because of $|xy| \leq p$, xy can only consist of 1s

According to the pumping lemma, we should have $xy^i z \in L$

However, by choosing i=0 we delete at least one 1 and get a word $w' = 1^{p-|y|}\, 0\, 2^p$ with $|y| \geq 1$.

w′ is not in L since it has fewer 1s than 2s.

This means that w is not pumpable and hence, L is not regular.