Discrete Event Systems

Exercise session #4

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1. Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

A) $L_1 = \{w \mid$ the length of w is odd$\}$

B) $L_2 = \{w \mid$ contains more 1s than 0s$\}$

1. Context-Free Grammars

A) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

1. Context-Free Grammars

A) L_1={w | the length of w is odd}

$$V = \{X, A\},$$
$$\Sigma = \{0, 1\},$$

$$R =$$

$$S = X$$

1. Context-Free Grammars

A) L_1={w | the length of w is odd}

$$V = \{X, A\},$$
$$\Sigma = \{0, 1\},$$
$$R = \left\{ \begin{aligned} &X \rightarrow XAX \mid A, \\ &A \rightarrow 0 \mid 1 \end{aligned} \right\},$$
$$S = X$$

1. Context-Free Grammars

B) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

1. Context-Free Grammars

B) L_2 = {w | contains more 1s than 0s}

$$S \rightarrow \; A1A$$
$$A \rightarrow \; AA \mid 1A0 \mid 0A1 \mid 1 \mid \varepsilon$$

1. Context-Free Grammars

B) L_2 = {w | contains more 1s than 0s}

$$S \rightarrow A1A$$
$$A \rightarrow AA \mid 1A0 \mid 0A1 \mid 1 \mid \varepsilon$$

2. Regular and Context-Free Languages

a) Consider the context-free grammar G with the production $S \rightarrow SS \mid 1S2 \mid 0$. Describe the language L(G) in words, and prove that L(G) is not regular.

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Assume L(G) is regular.

We take $w = 1^p 0\ 2^p \in L(G)$,

$w = xyz$ with $|xy| \leq p$ and $|y| \geq 1$, because of $|xy| \leq p$, xy can only consist of 1s

According to the pumping lemma, we should have $xy^i z \in L$

However, by choosing i=0 we delete at least one 1 and get a word $w' = 1^{p-|y|} 0\ 2^p$ with $|y| \geq 1$.

w′ is not in L(G) since it has fewer 1s than 2s.

This means that w is not pumpable and hence, L(G) is not regular.

2. Regular and Context-Free Languages

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

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b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

Since every regular language is also context-free, we can choose an arbitrary regular language.

$L = \{0^n1, n \geq 1\}$ is clearly regular.
A context-free grammar for this language uses only the production $S \rightarrow 0S \mid 1$.

3. Pumping Lemma Revisited

A) Determine whether the language $L = \{1^{n^2} \mid n \in N\}$ is regular.
Prove your claim!

3. Pumping Lemma Revisited

B) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number p to determine the number of states of a minimal DFA accepting L? What about the number of states of the corresponding NFA?

3. Pumping Lemma Revisited

$$\Sigma = \{a_1, a_2, ..., a_n\}$$

$$L = \bigcup_{i=1}^{n} a_i^* = a_1^* \cup a_2^* \cup \cdots \cup a_n^*$$

Minimum Pumping length 1,
but minimum DFA n+2