# Chapter 26

# Authenticated Agreement

In Section 18.4 we have already had a glimpse into the power of cryptography. In this Chapter we want to build a *practical* byzantine fault-tolerant system using cryptography. With cryptography, byzantine lies may be detected easily.

# 26.1 Agreement with Authentication

**Definition 26.1** (Signature). Every node can digitally **sign** its messages in a way that no other node can forge, thus nodes can reliably determine which node a signed message originated from. We denote a message msg(x) signed by node u with  $msg(x)_u$ .

#### Algorithm 26.2 Byzantine Agreement with Authentication

```
Code for primary p:
 1: if input is 1 then
      broadcast value(1)_p
      decide 1 and terminate
3:
      decide 0 and terminate
6: end if
    Code for all other nodes v:
7: for all rounds i \in \{1, \dots, f+1\} do
      S is the set of accepted messages value(1)<sub>u</sub>.
      if |S| \geq i and value(1)_p \in S then
        broadcast S \cup \{value(1)_v\}
        decide 1 and terminate
11:
      end if
12:
13: end for
14: decide 0 and terminate
```

• Algorithm 26.2 solves by zantine agreement on binary inputs relying on signatures. We assume there is a designated "primary" node p that all other nodes know. The goal is to decide on p's value.

**Theorem 26.3.** Algorithm 26.2 can tolerate f < n byzantine failures while terminating in f + 1 rounds.

*Proof.* Assuming that the primary p is not byzantine and its input is 1, then p broadcasts  $\mathtt{value}(1)_p$  in the first round, which will trigger all correct nodes to decide on 1. If p's input is 0, there is no signed message  $\mathtt{value}(1)_p$ , and no node can decide on 1.

If primary p is byzantine, we need all correct nodes to decide on the same value for the algorithm to be correct.

Assume i < f+1 is the minimal round in which any correct node u decides on 1. In this case, u has a set S of at least i messages from other nodes for value 1 in round i, including one of p. Therefore, in round  $i+1 \le f+1$ , all other correct nodes will receive S and u's message for value 1 and thus decide on 1 too.

Now assume that i=f+1 is the minimal round in which a correct node u decides for 1. Thus u must have received f+1 messages for value 1, one of which must be from a correct node since there are only f byzantine nodes. In this case some other correct node v must have decided on 1 in some round j < i, which contradicts i's minimality; hence this case cannot happen.

Finally, if no correct node decides on 1 by the end of round f+1, then all correct nodes will decide on 0.

#### Remarks:

- If the primary is a correct node, Algorithm 26.2 only needs two rounds! Otherwise, the algorithm terminates in at most f + 1 rounds, which is optimal as described in Theorem 17.20.
- By using signatures, Algorithm 26.2 manages to solve consensus for any number of failures! Does this contradict Theorem 17.12? Recall that in the proof of Theorem 17.12 we assumed that a byzantine node can distribute contradictory information about its own input. If messages are signed, correct nodes can detect such behavior. Specifically, if a node u signs two contradicting messages, then observing these two messages proves to all nodes that node u is byzantine.
- Does Algorithm 26.2 satisfy any of the validity conditions introduced in Section 17.1? No! A byzantine primary can dictate the decision value.
- Can we modify the algorithm such that the correct-input validity condition is satisfied? Yes! We can run the algorithm in parallel for 2f+1 primary nodes. Either 0 or 1 will occur at least f+1 times, which means that one correct process had to have this value in the first place. In this case, we can only handle  $f < \frac{n}{2}$  byzantine nodes.
- Can we make it work with arbitrary inputs?

• Relying on synchrony limits the practicality of the protocol. What if messages can be lost or the system is asynchronous?

## 26.2 Practical Byzantine Fault Tolerance

Practical Byzantine Fault Tolerance (PBFT) is one of the first and perhaps the most instructive protocol for achieving state replication among nodes as in Definition 15.8 with byzantine nodes in an asynchronous network. We present a simplified version of PBFT without any optimizations.

**Definition 26.4** (System Model). We consider a system with n=3f+1 nodes, and additionally an unbounded number of clients. There are at most f byzantine nodes, and clients can be byzantine as well. The network is asynchronous, and messages have variable delay and can get lost. Clients send requests that correct nodes have to order to achieve state replication.

#### Remarks:

- At any given time, every node will consider one designated node to be the *primary* and the other nodes to be *backups*.
- The timespan for which a node p is seen as the primary from the perspective of another node is called a view.

**Definition 26.5** (View). A view v is a non-negative integer representing the node's local perception of the system. We say that node u is in view v as long as node u considers node  $p = v \mod n$  to be the primary.

#### Remarks:

- All nodes start out in view 0. Nodes can potentially be in different views (i.e. have different local values for v) at any given time.
- If backups detect faulty behavior in the primary, they switch to the next primary with a so-called *view change* (see Section 26.4).
- In the asynchronous model, requests can arrive at the nodes in different orders. While a primary remains in charge (sufficiently many nodes share the view v), it thus adopts the function of a serializer (cf. Algorithm 15.9).

**Definition 26.6** (Sequence Number). During a view, a node relies on the primary to assign consecutive **sequence numbers** (integers) that function as indices in the global order (cf. Definition 15.8) for the requests that clients send.

#### Remarks:

• During a view change, we ensure that no two correct nodes execute requests in different orders. On the one hand, we need to exchange information on the current state to guarantee that a correct new primary knows the latest sequence number that has been accepted by sufficiently many backups. On the other hand, exchanging information will enable backups to determine if the new primary acts in a byzantine fashion, e.g. reassigning the latest sequence number to a different request.

• We use signatures to guarantee that every node can determine which node/client has generated any given message.

**Definition 26.7** (Accepted Messages). A correct node that is in view v will only **accept** messages that it can authenticate, that follow the specification of the protocol, and that also belong to view v.

#### Remarks:

- The protocol will guarantee that once a correct node has executed a request r with sequence number s, then no correct node will execute any request  $r' \neq r$  with sequence number s, not unlike Lemma 15.14.
- Correct primaries choose sequence numbers in order, without gap, i.e. if a correct primary proposed s as the sequence number for the last request, then it will use s+1 for the next request that it proposes.
- Before a node can safely execute a request r with a sequence number s, it will wait until it knows that the decision to execute r with s has been reached and is widely known.
- Informally, nodes will collect confirmation messages by sets of at least 2f+1 nodes to guarantee that that information is sufficiently widely distributed.

**Lemma 26.8** (2f+1 Quorum Intersection). Let  $S_1$  with  $|S_1| \ge 2f+1$  and  $S_2$  with  $|S_2| \ge 2f+1$  each be sets of nodes. Then there exists a correct node in  $S_1 \cap S_2$ .

*Proof.* Let  $S_1, S_2$  each be sets of at least 2f+1 nodes. There are 3f+1 nodes in total, thus due to the pigeonhole principle the intersection  $S_1 \cap S_2$  contains at least f+1 nodes. Since there are at most f faulty nodes,  $S_1 \cap S_2$  contains at least 1 correct node.

## 26.3 PBFT: Agreement Protocol

First we describe how PBFT achieves agreement on a unique order of requests within a single view. Figure 26.9 shows how the nodes come to an agreement on a sequence number for a client request. Informally, the protocol has these five steps:

- 1. The nodes receive a request and relay it to the primary.
- 2. The primary sends a pre-prepare-message to all backups, informing them that it wants to execute that request with the sequence number specified in the message.
- 3. Backups send prepare-messages to all nodes, informing them that they agree with that suggestion.
- 4. All nodes send commit-messages to all nodes, informing everyone that they have committed to execute the request with that sequence number.
- 5. They execute the request and inform the client.

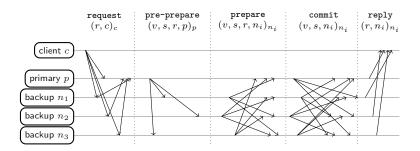


Figure 26.9: The agreement protocol used in PBFT for processing a client request issued by client c, exemplified for a system with n=4 nodes. The primary in view v is  $p=n_0=v \bmod n$ .

- To make sure byzantine nodes cannot force the execution of a request, every node waits for a certain number of prepare- and commitmessages with the correct content before executing the request.
- Definitions 26.10, 26.12, 26.14, and 26.16 specify the agreement protocol formally. Backups run the pre-prepare and the prepare phase concurrently.

**Definition 26.10** (Pre-Prepare Phase). In the **pre-prepare phase** of the agreement protocol, the nodes execute Algorithm 26.11.

### Algorithm 26.11 PBFT Agreement Protocol: Pre-Prepare Phase

Code for primary p in view v:

- 1: accept  $request(r, c)_c$  that originated from client c
- 2: pick next sequence number s
- 3: send pre-prepare $(v, s, r, p)_p$  to all backups

Code for backup b:

- 4: accept request $(r,c)_c$  from client c
- 5: relay request $(r, c)_c$  to primary p

**Definition 26.12** (Prepare Phase). In the **prepare phase** of the agreement protocol, every backup b executes Algorithm 26.13. Once it has sent the preparemessage, we say that b has **pre-prepared** r for (v, s).

#### Remarks:

• What if a byzantine primary does not send a pre-prepare message for a request?

#### Algorithm 26.13 PBFT Agreement Protocol: Prepare Phase

Code for backup b in view v:

- 1: accept pre-prepare $(v, s, r, p)_p$
- 2: if b has not yet accepted a pre-prepare-message for (v, s, r') with  $r' \neq r$  then
- 3: send prepare $(v, s, r, b)_b$  to all nodes
- 4: end if

**Definition 26.14** (Prepared-Certificate). A node  $n_i$  that has pre-prepared a request executes Algorithm 26.15. It waits until it has collected 2f preparemessages (including  $n_i$ 's own, if it is a backup) in Line 1. Together with the pre-prepare-message for (v, s, r), they form a prepared-certificate.

### Algorithm 26.15 PBFT Agreement Protocol: Commit Phase

Code for node  $n_i$  that has pre-prepared r for (v, s):

- 1: wait until 2f prepare-messages matching (v, s, r) have been accepted
- 2: create prepared-certificate for (v, s, r)
- 3: send commit $(v, s, n_i)_{n_i}$  to all nodes

**Definition 26.16** (Committed-Certificate). A node  $n_i$  that has created a prepared-certificate for a request executes Algorithm 26.17. It waits until it has collected 2f + 1 commit-messages (including  $n_i$ 's own) in Line 1. They form a committed-certificate and allow to safely execute the request once all requests with lower sequence numbers have been executed.

## Algorithm 26.17 PBFT Agreement Protocol: Execute Phase

Code for node  $n_i$  that has created a prepared-certificate for (v, s, r):

- 1: wait until 2f + 1 commit-messages matching (v, s) have been accepted
- 2: create committed-certificate for (v, s, r)
- 3: wait until all requests with lower sequence numbers have been executed
- 4: execute request r
- 5: send  $reply(r, n_i)_{n_i}$  to client

#### Remarks:

- Note that the agreement protocol can run for multiple requests in parallel. Since we are in the variable delay model and messages can arrive out of order, we thus have to wait in Algorithm 26.17 Line 3 for all requests with lower sequence numbers to be executed.
- The client only considers the request to have been processed once it received f + 1 reply-messages sent by the nodes in Algorithm 26.17 Line 5. Since a correct node only sends a reply-message once it executed the request, with f + 1 reply-messages the client can be certain that the request was executed by a correct node.

- We will see in Section 26.4 that PBFT guarantees that once a single correct node executed the request, then all correct nodes will never execute a different request with the same sequence number. Thus, knowing that a single correct node executed a request is enough for the client.
- If the client does not receive at least f+1 reply-messages fast enough, it can start over by resending the request to initiate Algorithm 26.11 again. To prevent correct nodes that already executed the request from executing it a second time, clients can mark their requests with some kind of unique identifiers like a local timestamp. Correct nodes can then react to each request that is resent by a client as required by PBFT, and they can decide if they still need to execute a given request or have already done so before.

**Lemma 26.18** (PBFT: Unique Sequence Numbers within View). If a node was able to create a prepared-certificate for (v, s, r), then no node can create a prepared-certificate for (v, s, r') with  $r' \neq r$ .

*Proof.* Assume two (not necessarily distinct) nodes create prepared-certificates for (v, s, r) and (v, s, r'). Since a prepared-certificate contains 2f + 1 messages, by Lemma 26.8, some correct node must have sent a pre-prepare- or preparemessage both matching (v, s, r) and (v, s, r'). However, a correct primary only sends a single pre-prepare-message for each (v, s), see Algorithm 26.11 Lines 2 and 3. Furthermore, a correct backup only sends a single prepare-message for each (v, s), see Algorithm 26.13 Lines 2 and 3. Thus, r' = r.

#### Remarks:

- Due to Lemma 26.18, once a node has a prepared-certificate for (v, s, r), no correct node will execute some  $r' \neq r$  with sequence number s during view v because correct nodes wait for a prepared-certificate before executing a request (cf. Algorithm 26.15).
- However, that is not yet enough to make sure that no  $r' \neq r$  will be executed by a correct node with sequence number s during some later view v' > v. How can we make sure that that does not happen?

## 26.4 PBFT: View Change Protocol

If the primary is faulty, the system has to perform a view change to move to the next primary so the system can make progress. To that end, nodes use a local faulty-timer (and only that!) to decide whether they consider the primary to be faulty.

**Definition 26.19** (Faulty-Timer). When backup b accepts request r in Algorithm 26.11 Line 4, b starts a local **faulty-timer** (if the timer is not already running) that will only stop once b executes r.

- If the faulty-timer expires, the backup considers the primary faulty and triggers a view change. When triggering a view change, a correct node will no longer participate in the protocol for the current view.
- We leave out the details regarding for what timespan to set the faultytimer. This is a patience trade-off (more patience: slower turnover if the primary is byzantine; less patience: risk of prematurely firing view changes).
- During a view change, the protocol has to guarantee that requests that have already been executed by some correct nodes will not be executed with the different sequence numbers by other correct nodes.
- How can we guarantee that this happens?

**Definition 26.20** (PBFT: View Change Protocol). In the view change protocol, a node whose faulty-timer has expired enters the view change phase by running Algorithm 26.22. During the new view phase (which all nodes continuously listen for), the primary of the next view runs Algorithm 26.24 while all other nodes run Algorithm 26.25.

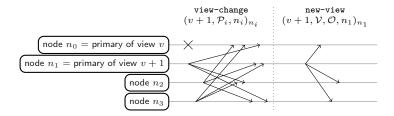


Figure 26.21: The view change protocol used in PBFT. Node  $n_0$  is the primary of current view v, node  $n_1$  the primary of view v+1. Once back-ups consider  $n_0$  to be faulty, they start the view change protocol (cf. Algorithms 26.22, 26.24, 26.25). The X signifies that  $n_0$  is faulty.

#### Remarks:

- The idea behind the view change protocol is as follows: during the view change protocol, the new primary collects prepared-certificates from 2f + 1 nodes, so for every request that some correct node executed, the new primary will have at least one prepared-certificate.
- After gathering that information, the primary distributes it and tells all backups which requests need to be to executed with which sequence numbers.
- Backups can check whether the new primary makes the decisions required by the protocol, and if it does not, then the new primary must be byzantine and the backups can directly move to the next view change.

#### Algorithm 26.22 PBFT View Change Protocol: View Change Phase

Code for node  $n_i$  in view v whose faulty-timer has expired:

- 1: stop accepting  ${\tt pre-prepare/prepare/commit-messages}$  for v
- 2: let  $\mathcal{P}_i$  be the set of all prepared-certificates that  $n_i$  has collected since the system was started
- 3: send view-change $(v+1, \mathcal{P}_i, n_i)_{n_i}$  to all nodes

**Definition 26.23** (New-View-Certificate). 2f + 1 view-change-messages for the same view v form a **new-view-certificate**.

#### Algorithm 26.24 PBFT View Change Protocol: New View Phase - Primary

Code for new primary p of view v + 1:

- 1: accept 2f + 1 view-change-messages (including possibly p's own) in a set  $\mathcal{V}$  (this is the new-view-certificate)
- 2: let  $\mathcal{O}$  be a set of  $\operatorname{\mathtt{pre-prepare}}(v+1,s,r,p)_p$  for all pairs (s,r) where at least one prepared-certificate for (s,r) exists in  $\mathcal{V}$
- 3: let  $s_{max}^{\mathcal{V}}$  be the highest sequence number for which  $\mathcal{O}$  contains a pre-prepare-message
- 4: add to  $\mathcal{O}$  a message pre-prepare $(v+1,s',\text{null},p)_p$  for every sequence number  $s' < s_{max}^{\mathcal{V}}$  for which  $\mathcal{O}$  does not contain a pre-prepare-message
- 5: send  $new-view(v+1, \mathcal{V}, \mathcal{O}, p)_p$  to all nodes
- 6: start processing requests for view v+1 according to Algorithm 26.11 starting from sequence number  $s_{max}^{V}+1$

#### Remarks:

• It is possible that  $\mathcal{V}$  contains a prepared-certificate for a sequence number s while it does not contain one for some sequence number s' < s. For each such sequence number s', we fill up  $\mathcal{O}$  in Algorithm 26.24 Line 4 with null-requests, i.e. requests that backups understand to mean "do not do anything here".

**Theorem 26.26** (PBFT:Unique Sequence Numbers Across Views). Together, the PBFT agreement protocol and the PBFT view change protocol guarantee that if a correct node executes a request r in view v with sequence number s, then no correct node will execute any  $r' \neq r$  with sequence number s in any view  $v' \geq v$ .

*Proof.* If no view change takes place, then Lemma 26.18 proves the statement. Therefore, assume that a view change takes place, and consider view v' > v.

We will show that if some correct node executed a request r with sequence number s during v, then a correct primary will send a pre-prepare-message matching (v', s, r) in the  $\mathcal{O}$ -component of the new-view $(v', \mathcal{V}, \mathcal{O}, p)$ -message. This guarantees that no correct node will be able to collect a prepared-certificate for s and a different  $r' \neq r$ .

Algorithm 26.25 PBFT View Change Protocol: New View Phase - Backup

```
Code for backup b of view v + 1 if b's local view is v' < v + 1:
 1: accept new-view(v+1, \mathcal{V}, \mathcal{O}, p)_p
 2: stop accepting pre-prepare-/prepare-/commit-messages for v
 3: set local view to v+1
 4: if p is primary of v+1 then
      if \mathcal{O} was correctly constructed from \mathcal{V} according to Algorithm 26.24
      Lines 2 and 4 then
        respond to all pre-prepare-messages in \mathcal{O} as in the agreement protocol,
 6:
        starting from Algorithm 26.13
        start accepting messages for view v+1
 7:
      else
 8:
        trigger view change to v + 2 using Algorithm 26.22
9:
      end if
10:
11: end if
```

Consider the new-view-certificate  $\mathcal{V}$  (see Algorithm 26.24 Line 1). If any correct node executed request r with sequence number s, then due to Algorithm 26.17 Line 1, there is a set  $R_1$  of at least 2f+1 nodes that sent a commit-message matching (s,r), and thus the correct nodes in  $R_1$  all collected a prepared-certificate in Algorithm 26.15 Line 1.

The new-view-certificate contains view-change-messages from a set  $R_2$  of 2f+1 nodes. Thus according to Lemma 26.8, there is at least one correct node  $c_r \in R_1 \cap R_2$  that both collected a prepared-certificate matching (s,r) and whose view-change-message is contained in  $\mathcal{V}$ .

Therefore, if some correct node executed r with sequence number s, then  $\mathcal{V}$  contains a prepared-certificate matching (s,r) from  $c_r$ . Thus, if some correct node executed r with sequence number s, then due to Algorithm 26.24 Line 2, a correct primary p sends a new-view $(v', \mathcal{V}, \mathcal{O}, p)$ -message where  $\mathcal{O}$  contains a pre-prepare(v', s, r, p)-message.

Correct backups will enter view v' only if the new-view-message for v' contains a valid new-view-certificate  $\mathcal V$  and if  $\mathcal O$  was constructed correctly from  $\mathcal V$ , see Algorithm 26.25 Line 5. They will then respond to the messages in  $\mathcal O$  before they start accepting other pre-prepare-messages for v' due to the order of Algorithm 26.25 Lines 6 and 7. Therefore, for the sequence numbers that appear in  $\mathcal O$ , correct backups will only send prepare-messages responding to the pre-prepare-messages found in  $\mathcal O$  due to Algorithm 26.13 Lines 2 and 3. This guarantees that in v', for every sequence number s that appears in  $\mathcal O$ , backups can only collect prepared-certificates for the triple (v',s,r) that appears in  $\mathcal O$ .

Together with the above, this proves that if some correct node executed request r with sequence number s in v, then no node will be able to collect a prepared-certificate for some  $r' \neq r$  with sequence number s in any view  $v' \geq v$ , and thus no correct node will execute r' with sequence number s.

- We have shown that PBFT protocol guarantees safety or nothing bad ever happens, i.e., the correct nodes never disagree on requests that were committed with the same sequence numbers. But, does PBFT also guarantee liveness? In other words, will a legitimate client request eventually be committed and replied?
- To prove liveness, we need message delays to be finite and bounded. With unbounded message delays in an asynchronous system and even one faulty process, it is impossible to solve consensus with guaranteed termination (Theorem 16.14).
- A faulty new primary could delay the system indefinitely by never sending a new-view-message. To prevent this, as soon as a node sends its view-change-message for view v+1, it starts its faulty-timer. and stops it once it accepts a for v+1. If the timer fires before receiving the new-view-message, the node triggers another view change.
- Since message delays are unknown, timers are doubling with every view. Eventually, the timeout is larger than the maximum message delay, and all correct messages are received before any timer expires.
- Since at most f consecutive primaries can be faulty, the system makes progress after at most f + 1 view changes.
- We described a simplified version of PBFT; any practically relevant variant makes adjustments to what we presented. The references found in the chapter notes can be consulted for details that we did not include.

# **Chapter Notes**

PBFT is perhaps the central protocol for asynchronous byzantine state replication. The seminal first publication about it, of which we presented a simplified version, can be found in [CL<sup>+</sup>99]. The canonical work about most versions of PBFT is Miguel Castro's PhD dissertation [Cas01].

Notice that the sets  $\mathcal{P}_b$  in Algorithm 26.22 grow with each view change as the system keeps running since they contain all prepared-certificates that nodes have collected so far. All variants of the protocol found in the literature introduce regular *checkpoints* where nodes agree that enough nodes executed all requests up to a certain sequence number so they can continuously garbage-collect prepared-certificates. We left this out for conciseness.

Remember that all messages are signed. Generating signatures is somewhat pricy, and variants of PBFT exist that use the cheaper, but less powerful Message Authentication Codes (MACs). These variants are more complicated because MACs only provide authentication between the two endpoints of a message and cannot prove to a third party who created a message. An extensive treatment of a variant that uses MACs can be found in [CL02].

Before PBFT, byzantine fault-tolerance was considered impractical, just something academics would be interested in. PBFT changed that as it showed

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that by zantine fault-tolerance can be practically feasible. As a result, numerous a synchronous by zantine state replication protocols were developed. Other well-known protocols are  $\rm Q/U~[AEMGG^+05], HQ~[CML^+06],$  and Zyzzyva [KAD^+07]. An overview over the relevant literature can be found in [AGK^+15].

This chapter was written in collaboration with Georg Bachmeier.

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