ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Digital Systems and Design Automation Group

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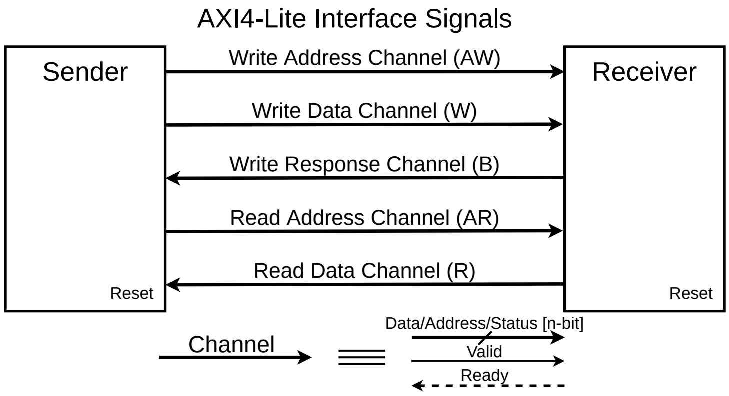
Prof. L. Josipović / J. Xu
based on Prof. L. Thiele's material

Discrete Event Systems

Exercise Sheet 11

1 Specifying Formal Properties Using Temporal Logic

The **Advanced eXtensible Interface (AXI)** is an on-chip bus protocol developed by ARM. In this exercise, we would like to derive a set of formal specifications using temporal logic for verifying that a newly developed HW module always interact with the bus in a correct way. In the following, we are interested in formulating the following properties in terms of temporal logic[1].



a) Liveness: each request (sender asserts a **valid**) in the channel should eventually be acknowledged (receiver asserts **ready**).

b) Fairness: for each channel, the receiver **ready** signal should assert infinitely often.

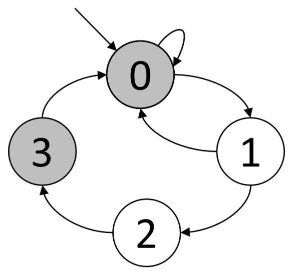
c) Persistency: for each channel, when the sender asserts its **valid** signal high, then it should be remained high until its respective **ready** is also high.

d) For each channel, the data signal must remain constant from when the **valid** signal is set, up until both **valid** and **ready** are set together.

[1]For the complete specification, you may visit the AMBA AXI and ACE Protocol Specification page.

2 Temporal Logic

a) We consider the following automaton. Property a holds in the colored states (0 and 3).



For each of the following CTL formula, list all the states for which it holds true.

(i) EF a

(ii) EG a

(iii) EX AX a

(iv) EF (a AND EX NOT(a))

b) You are given a set of states S, the transition function $f : S \mapsto S$, encoded as the characteristic function $\psi_f(q, q')$ (which returns *true* only if $f(q) = q'$), and the set $Z \subseteq S$ with its characteristic function $\phi_Z(q)$ (which returns *true* only if $q \in Z$). Your goal is to come up with a simple algorithm to find the characteristic function $\psi_{\text{AF } Z}(q)$, which encodes the set of all states satisfying AF Z.

(i) Give a relation between EG Z and AF Z.

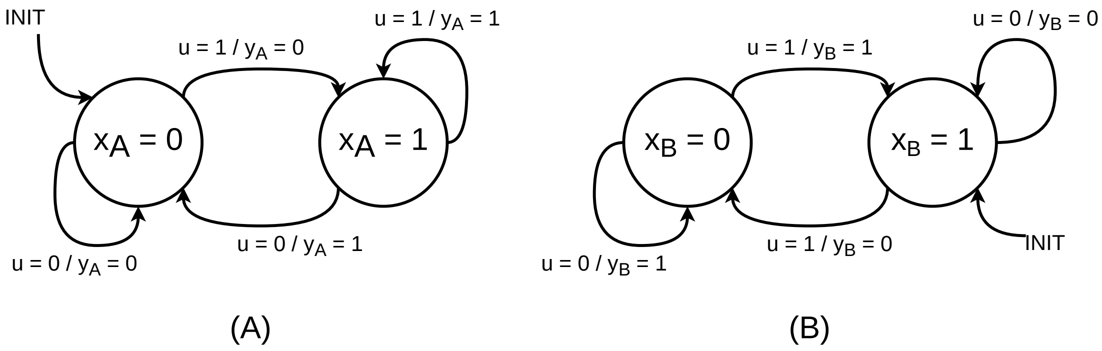
(ii) Using this relation, formulate an iterative procedure to find the set of states that satisfy AF Z. Use regular set operations to find the procedure. You can use the predecessor function $Pre(Q, f)$, which returns the set of states from which we can reach states in Q using the transition function f in one step.

$$Pre(Q, f) = \{q' : \exists q,\ \psi_f(q', q) \cdot \psi_Q(q) = 1\}$$

(iii) Translate the iteration procedure from (ii) into an algorithm using boolean expressions. Assume that you are given, for each set Q (i.e., its characteristic function), the characteristic function $\psi_{Pre(Q,f)}$.

3 Comparison of Finite Automata

Here are two simple finite automata:



For each automation, the state is represented using 1-bit encoding (x_A and x_B), one 1-bit output (y_A and y_B), and one common 1-bit input (u). We want to verify whether these two automata are equivalent. For this, we would like to investigate into the following steps:

a) Determine the characteristic function $\psi_A(x, x', u)$ and $\psi_B(x, x', u)$ of the transition relation for the two automata A and B.

b) Determine the characteristic function $\psi_f(x_A, x'_A, x_B, x'_B)$ of the transition relation for the product of the two automata.
Reminder: $\psi_f(x_A, x'_A, x_B, x'_B) := (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))$

c) Determine the characteristic function $\psi_X(x_A, x_B)$ of the set of reachable states of the product automata.

d) Determine the characteristic function $\psi_Y(y_A, y_B)$ of the set of reachable states of the product automata.

e) Are the two automata equivalent? **Hint:** Evaluate, for example, $\psi_Y(0, 1)$