Last week, we showed the equivalence of DFA, NFA and REX is equivalent to:

\[
\text{DFA} \simeq \text{NFA} \quad \text{(DFA and NFA are equivalent)}
\]

\[
\text{NFA} \simeq \text{REX} \quad \text{(NFA and REX are equivalent)}
\]
We also started to look at non-regular languages

Pumping lemma

If $A$ is a regular language, then there exist a number $p$ s.t.

Any string in $A$ whose length is at least $p$ can be divided into three pieces $xyz$ s.t.

- $xy^iz \in A$, for each $i \geq 0$ and
- $|y| > 0$ and
- $|xy| \leq p$
To prove that a language $A$ is not regular:

1. Assume that $A$ is regular

2. Since $A$ is regular, it must have a pumping length $p$

3. Find one string $s$ in $A$ whose length is at least $p$

4. For any split $s = xyz$,
   Show that you cannot satisfy all three conditions

5. Conclude that $s$ cannot be pumped
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1. Assume that $A$ is regular

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Pumping lemma

If \( A \) is a regular language, then there exist a number \( p \) s.t.

Any string in \( A \) whose length is at least \( p \) can be divided into three pieces \( xyz \) s.t.

- \( xy^iz \in A \), for each \( i \geq 0 \) and
- \( |y| > 0 \) and
- \( |xy| \leq p \)

Wait…
What happens if \( A \) is a finite language?!
Pumping lemma

If $A$ is a regular language, then there exist a number $p$ s.t.

As we saw two weeks ago, all finite languages are regular...

So what's $p$?

$p := \text{len(longest\_string)} + 1$

makes the lemma hold vacuously
Non-regular languages are not closed under most operations

if \( L_1 \) and \( L_2 \) are regular, then so are

\[
L_1 \cup L_2 \\
L_1 \cdot L_2 \\
L_1^*
\]

if \( L_1 \) and \( L_2 \) are \textbf{not regular}, then

\[
L_1 \cup L_2 \\
L_1 \cdot L_2 \\
L_1^*
\]

may or may not be regular!

\((L_1)^C\) is not regular

non RL are closed under complement
This week is all about

**Context-Free Languages**

a superset of Regular Languages