Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

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Computer Systems

Assignment 12

1 Game Theory

Quiz

1.1 Selling a Franc

Form groups of two to three people. Every member of the group is a bidder in an auction for one (imaginary) franc. The franc is allocated to the highest bidder (for his/her last bid). Bids must be a multiple of CHF 0.05. This auction has a crux. Every bidder has to pay the amount of money he/she bid (last bid) – it does not matter if he/she gets the franc. Play the game!

a) Where did it all go wrong?

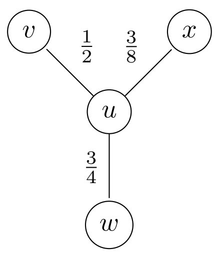
b) What could the bidders have done differently?

Basic

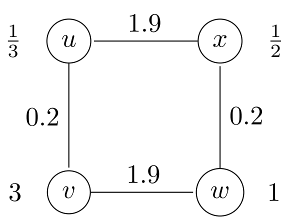
1.2 Selfish Caching

For each of the following caching networks, compute the social optimum, the pure Nash equilibria, the price of anarchy (PoA) as well as the optimistic price of anarchy($OPoA$):

i. $d_u = d_v = d_w = d_x = 1$



ii. The demand is written next to a node.



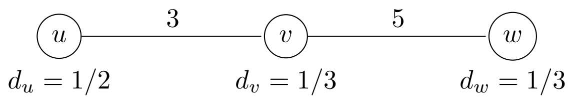
1.3 Selfish Caching with variable caching cost

The selfish caching model introduced in the lecture assumed that every peer incurs the same caching cost. However, this is a simplification of the reality. A peer with little storage space could experience a much higher caching cost than a peer who has terabytes of free disc space available. In this exercise, we omit the simplifying assumption and allow variable caching costs α_i for node i.

What are the Nash Equilibria in the following caching networks given that

i. $\alpha_u = 1$, $\alpha_v = 2$, $\alpha_w = 2$,

ii. $\alpha_u = 3$, $\alpha_v = 3/2$, $\alpha_w = 3$?



Does any of the above instances have a dominant strategy profile? What is the PoA of each instance?

Advanced

1.4 Matching Pennies

Tobias and Stephan like to gamble, and came up with the following game: Each of them secretly turns a penny to heads or tails. Then they reveal their choices simultaneously. If the pennies match Tobias gets both pennies, otherwise Stephan gets them.

Write down this 2-player game as a bi-matrix, and compute its (mixed) Nash equilibria!

1.5 PoA Classes

The PoA of a class $\mathcal{C}$ is defined as the maximum PoA over all instances in $\mathcal{C}$. Let

- $\mathcal{A}^n_{[a,b]}$ be the class of caching networks with n peers, $a \leq \alpha_i \leq b$, $d_i = 1$, and each edge has weight 1,
- $\mathcal{W}^n_{[a,b]}$ be the class of networks with n peers, $a \leq d_i \leq b$, $\alpha_i = 1$, and each edge has weight 1.

Show that $PoA(\mathcal{A}^n_{[a,b]}) \leq \frac{b}{a} \cdot PoA(\mathcal{W}^n_{[\frac{1}{b},\frac{1}{a}]})$ for all $n > 0$.

2 Quorum Systems

Quiz

2.1 The Resilience of a Quorum System

a) Does a quorum system exist, which can tolerate that all nodes of a specific quorum fail? Give an example or prove its nonexistence.

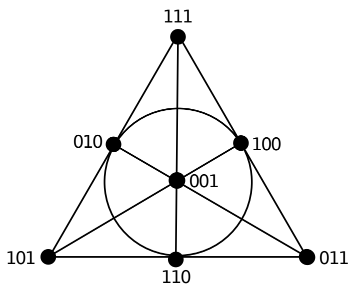
b) Consider the *nearly all* quorum system, which is made up of n different quorums, each containing $n-1$ servers. What is the resilience of this quorum system?

c) Can you think of a quorum system that contains as many quorums as possible?
Note: the quorum system does not have to be minimal.

Basic

2.2 A Quorum System

Consider a quorum system with 7 nodes numbered from 001 to 111, in which each three nodes fulfilling $x \oplus y = z$ constitute a quorum. In the following picture this quorum system is represented: All nodes on a line (such as 111, 010, 101) and the nodes on the circle (010, 100, 110) form a quorum.



a) Of how many different quorums does this system consist of and what are its work and its load?

b) Calculate its resilience f. Give an example where this quorum system does not work anymore with $f+1$ faulty nodes.

Advanced

2.3 Uniform Quorum Systems

Definitions:

s-Uniform: A quorum system $\mathcal{S}$ is *s-uniform* if every quorum in $\mathcal{S}$ has exactly s elements.
Balanced access strategy: An access strategy Z for a quorum system $\mathcal{S}$ is *balanced* if it satisfies $L_Z(v_i) = L$ for all $v_i \in V$, for some value L.

Claim: An s-uniform quorum system $\mathcal{S}$ reaches an optimal load with a balanced access strategy, if such a strategy exists.

a) Describe in your own words why this claim is true.

b) Prove the optimality of a balanced access strategy on an s-uniform quorum system.