Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

DYNAMO

HS 2023

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based on Prof. Lothar Thiele's material

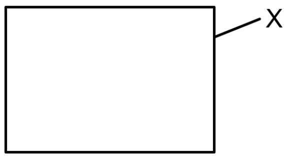
Discrete Event Systems

Solution to Exercise Sheet 11

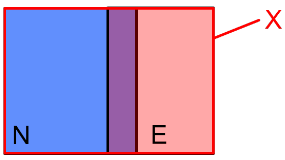
1 Set Representation

1.1 Warm-up

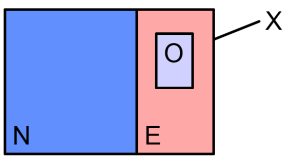
a) $\psi_X = 1$



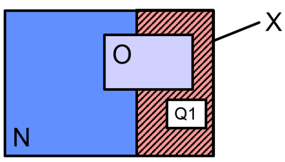
b) $N \cup E = X \quad \Leftrightarrow \quad \psi_N + \psi_E = 1$



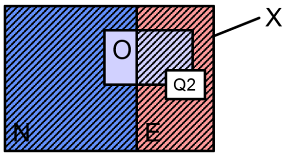
c) $N \cap O = \emptyset \quad \Leftrightarrow \quad \psi_N \cdot \psi_O = 0$



d) $Q_1 = E \backslash O \quad \Leftrightarrow \quad \psi_{Q_1} = \psi_E \cdot \overline{\psi_O}$



e) $$\begin{aligned} Q_2 = (O \cap E) \cup \overline{O} &= (O \cup \overline{O}) \cap (E \cup \overline{O}) \quad \Leftrightarrow \quad \psi_{Q_2} = \psi_E + \overline{\psi_O} \\ &= X \cap (E \cup \overline{O}) \\ &= E \cup \overline{O} \end{aligned}$$



1.2 Specification Composition

a) The specification for **C1**, **C2** and **C3** are the following:

C1 $\psi_{C1} = (x_1 + x_2 + x_3) \rightarrow x_s$

$$\psi_{C1} = (x_1 + x_2 + x_3)x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} = x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$$

C2 $\psi_{C2} = x_1 \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_1} \cdot x_2 \cdot \overline{x_3} + \overline{x_1} \cdot \overline{x_2} \cdot x_3 + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$

C3 $\psi_{C3} = x_b \rightarrow (x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3})$

$$\psi_{C3} = x_b \cdot x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b} = x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b}$$

b) The specification consists in satisfying all constraints at all times:

$$\psi_N = \psi_{C1} \cdot \psi_{C2} \cdot \psi_{C3}$$

2 Binary Decision Diagrams

2.1 Verification using BDDs

a) $f_2 : y = \overline{\overline{x_1 + x_2 + x_3} + \overline{x_1 + \overline{x_2} + \overline{x_3}} + \overline{\overline{x_1} + \overline{x_2} + x_3}}$

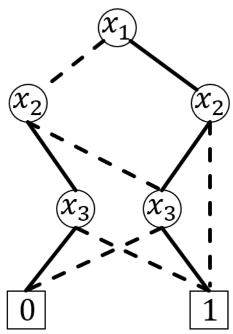
b) for f_1, we have

- case $x_1 = 0$:
  $y_{|x_1=0} = \overline{x_2}x_3 + x_2\overline{x_3}$
  - case $x_2 = 0$:
    $y_{|x_1=0,x_2=0} = x_3$
  - case $x_2 = 1$:
    $y_{|x_1=0,x_2=1} = \overline{x_3}$
- case $x_1 = 1$:
  $y_{|x_1=1} = \overline{x_2} + x_3 + \overline{x_2}x_3$
  - case $x_2 = 0$:
    $y_{|x_1=1,x_2=0} = 1$
  - case $x_2 = 1$:
    $y_{|x_1=1,x_2=1} = x_3$

for f_2, we have

- case $x_1 = 0$:
  $y_{|x_1=0} = \overline{\overline{x_2 + x_3} + \overline{\overline{x_2} + \overline{x_3}}}$
  - case $x_2 = 0$:
    $y_{|x_1=0,x_2=0} = \overline{\overline{x_3} + \overline{1 + \overline{x_3}}} = x_3$
  - case $x_2 = 1$:
    $y_{|x_1=0,x_2=1} = \overline{\overline{1} + \overline{\overline{x_3}}} = \overline{x_3}$
- case $x_1 = 1$:
  $y_{|x_1=1} = \overline{\overline{1} + \overline{1} + \overline{\overline{x_2} + x_3}} = \overline{x_2} + x_3$
  - case $x_2 = 0$:
    $y_{|x_1=1,x_2=0} = 1$
  - case $x_2 = 1$:
    $y_{|x_1=1,x_2=1} = x_3$

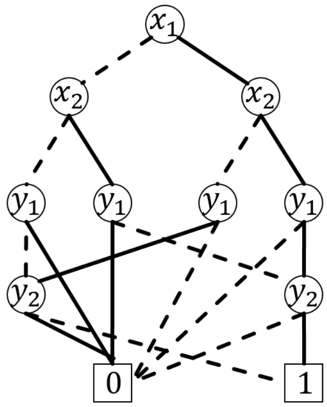
The two ROBDDs have identical falls, therefore they are equivalent.



2.2 BDDs with Respect to Different Orderings

a) $g = x_1\Big\{x_2\big[y_1(y_2)+\overline{y_1}(0)\big]+\overline{x_2}[y_1(\overline{y_2})+\overline{y_1}(0)]\Big\}+\overline{x_1}\Big\{x_2\big[y_1(0)+\overline{y_1}(y_2)\big]+\overline{x_2}\big[y_1(0)+\overline{y_1}(\overline{y_2})\big]\Big\}$

b) The ROBDD for g is the following:



c) Using the new ordering π', the Boole-Shannon decomposition becomes

$$g = x_1\Big\{y_1\big[x_2(y_2) + \overline{x_2}(\overline{y_2})\big] + \overline{y_1}[0]\Big\} + \overline{x_1}\Big\{y_1[0] + \overline{y_1}\big[x_2(y_2) + \overline{x_2}(\overline{y_2})\big]\Big\}.$$

This is a better ordering as it leads to a ROBDD with fewer nodes with respect to π (6 instead of 9).