ETH
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Distributed Computing

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Computational Thinking
Solutions to Exercise 6 (Cryptography)

1 Zero Knowledge Proofs in Geometry

a) The constructions are simple and we show here for example how to bisect an angle. First, we fix the compass to an arbitrary opening and draw a circle around the tip of the angle. We label the intersection points of the circle and the sides of the angle as A and B. We draw circles around points A and B with radii AB. Finally, we construct a line between the tip of the angle and the intersection of the (newly constructed) circles.[1]

b) The following example is one of the possible protocols:

ZKP in Geometry

Peggy		**Vic**
knows $\alpha, \beta = 3\alpha$		knows β
create random angle γ		
construct $\tau = 3\gamma$	send over τ $\longrightarrow$	
	$\longleftarrow$ send over c	choose randomly $c \in \{0, 1\}$
create $\rho = \gamma + c\alpha$	send over ρ $\longrightarrow$	check $3\rho \stackrel{?}{=} \tau + c\beta$

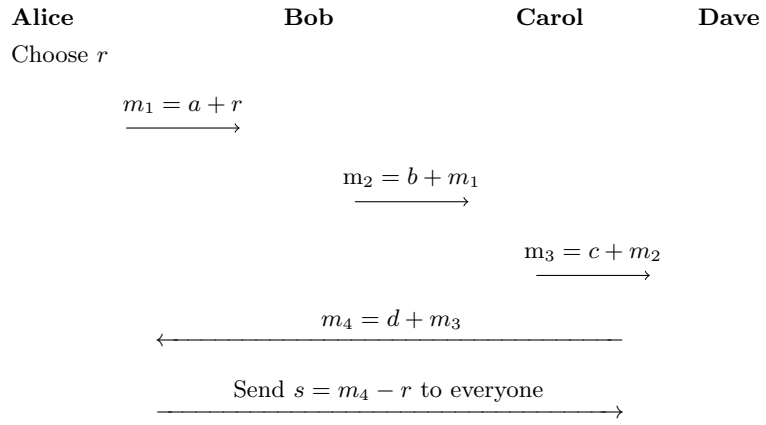
- **Completeness.** One can easily see that if Peggy is honest and knows α, Vic always accepts. More concretely, in the last step $3\rho = 3(\gamma + c\alpha) = 3\gamma + 3c\alpha = \tau + c\beta$.
- **Soundness.** We show that if Peggy can answer both challenges then she knows α. Assume Peggy can answer for both challenges $c = 0$ and $c' = 1$ correctly with $\rho = \gamma + 0 \cdot \alpha = \gamma$ and $\rho' = \gamma + \alpha$. Then it follows that Peggy can compute $\alpha = \rho' - \rho$. In other words, if she doesn't know α she can answer at most one of the challenges, and fail at the other challenge. That is, Peggy can correctly answer in one round only with probability $1/2$, and therefore n rounds only with probability $1/2^n$.
- **Zero Knowledge.** Vic cannot convince a third party that Peggy knows α. The main idea is to show that the same transcript that Vic has after the protocol could be generated by himself (without knowing α). During the protocol, the transcript contains the triples (τ, c, ρ) and can be produced as follows. For each challenge c, generate a random ρ and construct $\tau = 3\rho - c\beta$. To show zero-knowledge in general we need to show that the transcript can be generated for any strategy V' which is out of scope for this lecture.

[1] You may have a look at the following video about these constructions and the impossibility of trisecting an angle, if desired: `https://youtu.be/O1sPvUr0YC0`

2 MPC with Secret Sharing

a) One can easily see that each party after summing locally holds a share of the polynomial $f(x) = f_1(x) + f_2(x)$ (since the degree of the polynomial by summing doesn't change and t points uniquely define a polynomial of degree $t-1$). It follows that $s = f(0) = f_1(0)+f_2(0) = s_1 + s_2$. Hence, if the polynomial is reconstructed and evaluated at point 0, it will give the sum of s_1 and s_2.

b) Dave can just continue the same way as other participants:



c) Alice and Carol can easily compute Bob's salary as follows. Since Carol has $m_2 = b + m_1$ and Alice has m_1 they can compute $b = m_2 - m_1$. The same technique can be used for Dave as well, namely $d = m_4 - m_3$.

d) The main idea is to use secret sharing and its linearity property. Each party shares their salary by using (n, n) Shamir secret sharing, compute locally the sum of each share (of each salary), and in the end reconstruct the sum only.

More concretely, initially, each party has a (public) number and a secret salary. For example, Alice has $(1, s_A)$, Bob has $(2, s_B)$, Carol has $(3, s_C)$ and Dave has $(4, s_D)$. Each party generates a random polynomial to share their secret salary:

$$p_A(x) = s_A + a_1x + a_2x^2 + a_3x^3 \qquad \text{random polynomial of Alice}$$
$$p_B(x) = s_B + b_1x + b_2x^2 + b_3x^3 \qquad \text{random polynomial of Bob}$$
$$p_C(x) = s_C + c_1x + c_2x^2 + c_3x^3 \qquad \text{random polynomial of Carol}$$
$$p_D(x) = s_D + d_1x + d_2x^2 + d_3x^3 \qquad \text{random polynomial of Dave}$$

Then they distribute the shares of each secret and sum all the shares they receive (sum of columns in the following table):

	to Alice	to Bob	to Carol	to Dave
Alice sends	$p_A(1)$	$p_A(2)$	$p_A(3)$	$p_A(4)$
Bob sends	$p_B(1)$	$p_B(2)$	$p_B(3)$	$p_B(4)$
Carol sends	$p_C(1)$	$p_C(2)$	$p_C(3)$	$p_C(4)$
Dave sends	$p_D(1)$	$p_D(2)$	$p_D(3)$	$p_D(4)$
Sum locally	sum_A	sum_B	sum_C	sum_D

Now everyone broadcasts to everyone what the summation of the shares is. For example, Alice sends to everyone sum_A publicly. So do Bob, Carol and Dave. With those values $(sum_A, sum_B, sum_C, sum_D)$ everyone can reconstruct the polynomial (by Lagrange) and evaluate the polynomial at $x = 0$, which is the sum of the salaries.