

# Discrete Event Systems

## Exercise Sheet 4

### 1 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet  $\Sigma = \{0, 1\}$ :

- a)  $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$
- b)  $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

### 2 Regular and Context-Free Languages

- a) Consider the context-free grammar  $G$  with the production  $S \rightarrow SS \mid 1S2 \mid 0$ . Describe the language  $L(G)$  in words, and prove that  $L(G)$  is not regular.
- b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language  $L$  that is regular.

### 3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts  $L$ .

- a)  $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} = u\} = \{u \mid \text{"}u \text{ is a palindrome"}\}$
- b)  $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} \neq u\} = \{u \mid \text{"}u \text{ is no palindrome"}\}$

### 4 Ambiguity

Consider the following context-free grammar  $G$  with non-terminals  $S$  and  $A$ , start symbol  $S$ , and terminals “(”, “)”, and “0”:

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

- a) What are the eight shortest words produced by  $G$ ?
- b) Context-free grammars can be ambiguous. Prove or disprove that  $G$  is unambiguous.
- c) Design a push-down automaton  $M$  that accepts the language  $L(G)$ . If possible, make  $M$  deterministic.

## 5 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter*  $C$ , i.e., a register that can hold a single integer of arbitrary size. Initially,  $C = 0$ . We call such an automaton a *Counter Automaton*  $M$ .  $M$  can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let  $\mathcal{L}_{count}$  be the set of languages recognized by counter automata.

- a) Let  $\mathcal{L}_{reg}$  be the set of regular languages. Prove that  $\mathcal{L}_{reg} \subseteq \mathcal{L}_{count}$ .
- b) Prove that the opposite is not true, that is,  $\mathcal{L}_{count} \not\subseteq \mathcal{L}_{reg}$ . Do so by giving a language which is in  $\mathcal{L}_{count}$ , but not in  $\mathcal{L}_{reg}$ . Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.
- c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.

## 6 Inequality-checking with PDAs [Exam HS20]

Draw a PDA that recognizes  $L = \{x\#y \mid x, y \in \{0, 1\}^+, x \neq y\}$ . Use at most 12 states.

*Hint: Can two strings  $x$  and  $y$  with  $|x| < |y|$  ever be equal?*