

Discrete Event Systems

Exercise Sheet 14

1 Time Petri Net

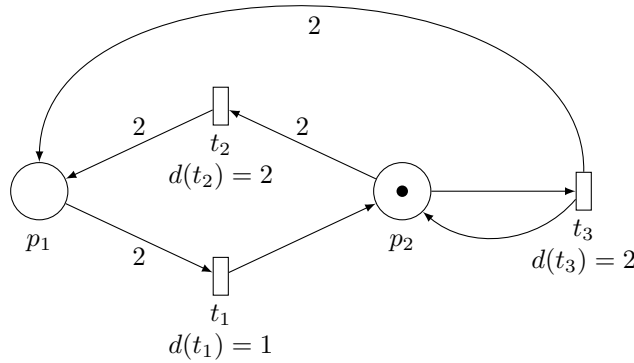


Figure 1: Time Petri net at simulation step 0 ($\tau = 0$).

Consider the Petri net in Figure 1. The transitions are associated with the following delays between their activation and firing: $d(t_1) = 1$, $d(t_2) = 2$, $d(t_3) = 2$. Simulate the behavior of the time Petri net by filling in the table below. For each simulated step, corresponding to a firing of the Petri net, indicate the simulation time τ , the transition t_{fired} that fires in τ , the resulting marking M^τ , and the updated event list L^τ . The first two simulation steps are already indicated in the table.

Note: If there are several transitions enabled at the same time, they fire in the order of their index, i.e., the transition with the smallest index fires first.

step	τ	t_{fired}	M^τ	L^τ
0	0	-	$[0, 1]$	$(t_3, 2)$
1	2	t_3	$[2, 1]$	$(t_1, 3), (t_3, 4)$
2				
3				
4				
5				

2 Liveness Properties in Petri Net

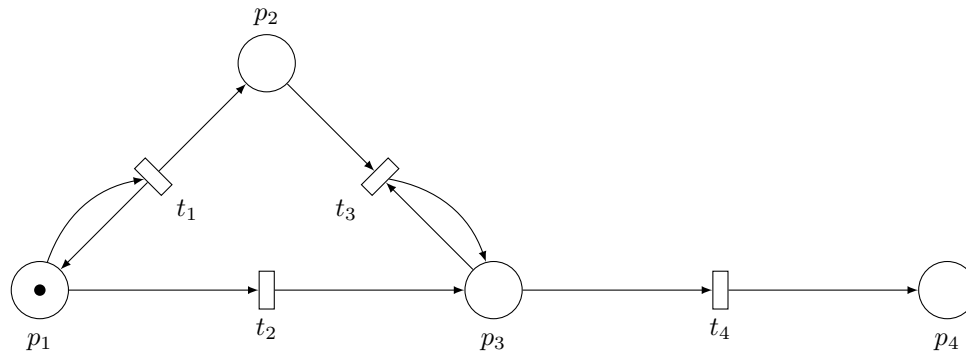


Figure 2: A Petri net.

Consider the Petri net depicted in Figure 2. Determine the highest liveness level for transitions t_1 , t_2 , t_3 , and t_4 . **Justify your answer** for each liveness level that you determine; if the transition is L_i live but not L_{i+1} -live, explain why.

Note: A transition t in a Petri net is

- dead iff t cannot be fired in any firing sequence,
- L_1 -live iff t can be fired at least once in some firing sequence,
- L_2 -live iff, $\forall k \in \mathbb{N}^+$, t can be fired at least k times in some firing sequence,
- L_3 -live iff t appears infinitely often in some infinite firing sequence,
- L_4 -live iff t is L_1 live for every marking that is reachable from M_0 .

L_{j+1} liveness implies L_j liveness.

3 (Bonus) Calculating with Petri nets

In this exercise you are supposed to model a function $f_i(x, y)$ using a Petri net. That is, the Petri net must contain two places P_x and P_y that hold x and y tokens respectively in the beginning. Additionally, the net must contain one place P_z which holds $f_i(x, y)$ tokens when the net is dead. The Petri nets are supposed to work for arbitrary numbers of tokens in P_x and P_y .

- a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$
- b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$
- c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$ Here you may want to use inhibitor arcs. An inhibitor arc between a place and a transition prevents the transition from firing as long as there is at least one token in the place.

Hint Start by creating a net that "duplicate" the number of tokens from P_x in place P_z . Then adapt this net to perform the multiplication.