Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

DYNAMO

HS 2024

Prof. Dr. Lana Josipović and Jiahui Xu
based on Prof. Dr. Lothar Thiele's material

# Discrete Event Systems

## Solution to Exercise Sheet 11

## 1 Set Representation

### 1.1 Set Operations and Characteristic Functions

**a)** Characteristic function: $\psi_X = 1$.

**b)** Set notation: $N \cup E = X$. Characteristic function: $\psi_N + \psi_E = 1$.

**c)** Set notation: $N \cap O = \emptyset$. Characteristic function: $\psi_N \cdot \psi_O = 0$.

**d)** Set notation: $Q_1 = E \setminus O$. Characteristic function: $\psi_{Q_1} = \psi_E \cdot \overline{\psi_O}$.

**e)** Set notation:
$$Q_2 = (O \cap E) \cup \overline{O} = (O \cup \overline{O}) \cap (E \cup \overline{O}) = X \cap (E \cup \overline{O}) = E \cup \overline{O}$$

Characteristic function: $\psi_{Q_2} = \psi_E + \overline{\psi_O}$.

### 1.2 Write Specifications in Boolean Encoding and Compose Them

**a)** The specification for **C1**, **C2** and **C3** described using characteristic functions are the following:

**C1** : $\psi_{C1} = (x_1 + x_2 + x_3) \rightarrow x_s$ $\psi_{C1} = (x_1 + x_2 + x_3)x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} = x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$.

**C2** : $\psi_{C2} = x_1 \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_1} \cdot x_2 \cdot \overline{x_3} + \overline{x_1} \cdot \overline{x_2} \cdot x_3 + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$.

**C3** : $\psi_{C3} = x_b \rightarrow (x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3})$.

$\psi_{C3} = x_b \cdot x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b} = x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b}$.

**b)** The specification consists in satisfying all constraints at all times:

$$\psi_N = \psi_{C1} \cdot \psi_{C2} \cdot \psi_{C3}.$$

# 2 Sets of States and State Transitions

See solution in the written exam of HS2023.

# 3 Binary Decision Diagrams

## 3.1 Verifying the Equivalence of Combinational Circuits Using BDDs

**a)** $f_2 : y = \overline{\overline{x_1 + x_2 + x_3} + \overline{x_1 + \overline{x_2} + \overline{x_3}} + \overline{\overline{x_1} + \overline{x_2} + x_3}}$

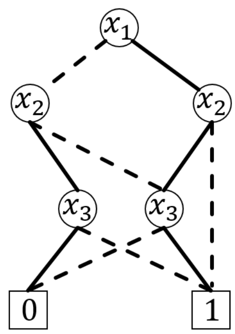
**b)** for $f_1$, we have

- case $x_1 = 0$:
  $y_{|x_1=0} = \overline{x_2}x_3 + x_2\overline{x_3}$
  - case $x_2 = 0$:
    $y_{|x_1=0,x_2=0} = x_3$
  - case $x_2 = 1$:
    $y_{|x_1=0,x_2=1} = \overline{x_3}$
- case $x_1 = 1$:
  $y_{|x_1=1} = \overline{x_2} + x_3 + \overline{x_2}x_3$
  - case $x_2 = 0$:
    $y_{|x_1=1,x_2=0} = 1$
  - case $x_2 = 1$:
    $y_{|x_1=1,x_2=1} = x_3$

for $f_2$, we have

- case $x_1 = 0$:
  $y_{|x_1=0} = \overline{\overline{x_2 + x_3} + \overline{\overline{x_2} + \overline{x_3}}}$
  - case $x_2 = 0$:
    $y_{|x_1=0,x_2=0} = \overline{\overline{x_3} + \overline{1 + \overline{x_3}}} = x_3$
  - case $x_2 = 1$:
    $y_{|x_1=0,x_2=1} = \overline{\overline{1} + \overline{\overline{x_3}}} = \overline{x_3}$
- case $x_1 = 1$:
  $y_{|x_1=1} = \overline{\overline{1} + \overline{1} + \overline{\overline{x_2} + x_3}} = \overline{x_2} + x_3$
  - case $x_2 = 0$:
    $y_{|x_1=1,x_2=0} = 1$
  - case $x_2 = 1$:
    $y_{|x_1=1,x_2=1} = x_3$

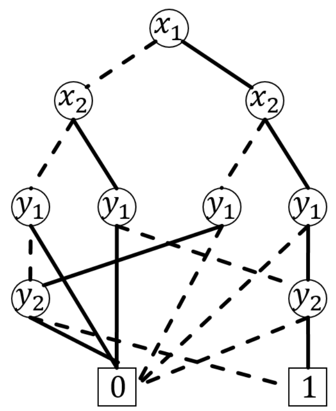
The two ROBDDs have identical falls, therefore they are equivalent.



### 3.2 ROBDDs with Respect to Different Orderings

**a)** $g = x_1\Big\{x_2\big[y_1(y_2)+\overline{y_1}(0)\big]+\overline{x_2}[y_1(\overline{y_2})+\overline{y_1}(0)]\Big\}+\overline{x_1}\Big\{x_2\big[y_1(0)+\overline{y_1}(y_2)\big]+\overline{x_2}\big[y_1(0)+\overline{y_1}(\overline{y_2})\big]\Big\}$

**b)** The ROBDD for $g$ is the following:




**c)** Using the new ordering $\pi'$, the Boole-Shannon decomposition becomes

$$g = x_1\Big\{y_1\big[x_2(y_2) + \overline{x_2}(\overline{y_2})\big] + \overline{y_1}[0]\Big\} + \overline{x_1}\Big\{y_1[0] + \overline{y_1}\big[x_2(y_2) + \overline{x_2}(\overline{y_2})\big]\Big\}.$$

This is a better ordering as it leads to a ROBDD with fewer nodes with respect to $\pi$ (6 instead of 9).