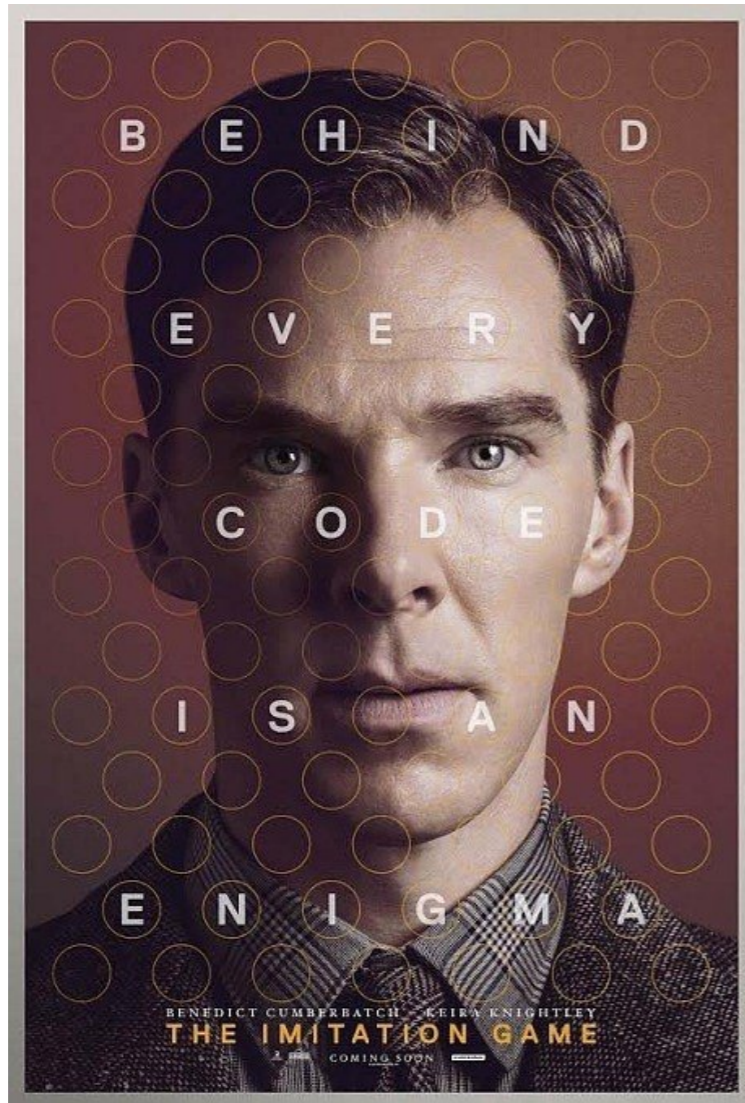# Automata & languages

A primer on the Theory of Computation

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Part 4 out of 5

Last week, we showed the
equivalence of DFA, NFA and REX

is equivalent to

DFA $\asymp$ NFA

$)($

REX

We also started to look at

non-regular languages

If A is a regular language, then

there exist a number p s.t.

Any string in A whose length is at least p

can be divided into three pieces xyz s.t.

- $xy^iz \in A$, for each i≥0 and
- $|y| > 0$ and
- $|xy| \leq p$

To prove that a language *A* is not regular:

1   Assume that *A* is regular

2   Since *A* is regular, it must have a pumping length p

3   Find one string s in *A* whose length is at least p

4   For any split *s=xyz,*
    Show that you cannot satisfy all three conditions

5   Conclude that s cannot be pumped

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3   Find one string *s* in *A* whose length is at least *p*

4   For any split *s=xyz,*
       Show that you cannot satisfy all three conditions

5   Conclude that **s cannot be pumped** ⟶ **A is not regular**

Wait...

What happens if A is a finite language?!

Pumping lemma

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there exist a number p s.t.

Any string in A whose length is at least p

can be divided into three pieces xyz s.t.

- $xy^iz \in A$, for each i≥0 and
- $|y| > 0$ and
- $|xy| \leq p$

Pumping lemma

If **_A_ is a regular language**, then there exist a number p s.t.

As we saw two weeks ago, **all finite languages** are regular…

So what's **p?**

p := len(longest_string) + 1

makes the lemma hold vacuously

Non-regular languages are not closed under most operations

if L_1 and L_2 are regular,
then so are

$L_1 \cup L_2$

$L_1 . L_2$

L_1^*

if L_1 and L_2 are not regular,
then

$L_1 \cup L_2$

$L_1 . L_2$ may or may not be
regular!

L_1^*

$(L_1)^C$ is not regular

non RL are closed under complement

This week is all about

Context-Free Languages

a superset of Regular Languages