Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Principles of Distributed Computing
Exercise 6

1 Maximal vs. Maximum Matching

Let M be a maximal matching and M^* be a maximum matching of a graph $G = (V, E)$. Prove the following two statements.

a) $\frac{|M^*|}{2} \leq |M| \leq |M^*|$.

b) $|M| \geq \frac{|E|}{2\Delta - 1}$.

2 Maximal Matching in Bipartite Graphs

Provide an algorithm that finds a maximal matching in a 2-colored bipartite graph in 2Δ rounds.

3 Maximal Matching using Forest Decomposition

a) Recall the 3-coloring algorithm for rooted trees in $O(\log^* n)$ time. Show that the algorithm can be adapted to work in $O(\log^* q)$ time if a q-coloring of the tree is provided.

b) Provide an algorithm that decomposes a graph with maximum degree Δ into at most Δ many edge-disjoint forests in $O(1)$ time.

c) Suppose that each of the forests is 3-colored. Provide an algorithm that runs in 3Δ rounds and finds a maximal matching in G.

d) Devise an algorithm that finds a maximal matching in $O(\Delta + \log^* q)$ time in a q-colored graph with maximum degree Δ.

4 Rounding in Non-Bipartite Graphs

Provide an example of a 2-decomposition of a non-bipartite graph in which one rounding step (rounding by a factor 2) leads to a loss $> \frac{1}{\log \Delta}$.