Network Decompositions

**Exercise 1:** Explain how given a $(C,D)$ network decomposition of graph $G$, a maximal independent set can be computed in $O(CD)$ rounds.

**Exercise 2:** We here see that the $(O(\log n), O(\log n))$ network decomposition that we discussed in the class has the nearly best possible parameters. In particular, it is known that there are $n$-node graphs that have girth\(^1\) $\Omega(\log n / \log \log n)$ and chromatic number $\Omega(\log n)$ [AS04, Erd59]. Use this fact to argue that on these graphs, an $(o(\log n), o(\log n / \log \log n))$ network decomposition does not exist.

**Exercise 3:** Given an $n$-node undirected graph $G = (V,E)$, we define a $d(n)$-diameter ordering of $G$ to be a one-to-one labeling $f : V \to \{1, 2, \ldots, n\}$ of vertices such that for any path $P = v_1, v_2, \ldots, v_p$ on which the labels $f(v_i)$ are monotonically increasing, any two nodes $v_i, v_j \in P$ have $\text{dist}_G(v_i, v_j) \leq d(n)$.

Use the existence of $(O(\log n), O(\log n))$ network decompositions, proved in the class, to argue that each $n$-node graph has an $O(\log^2 n)$-diameter ordering.

**References**


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\(^1\)Recall that the girth of a graph is the length of its shortest cycle.