Fast and Robust GPS Fix Using One Millisecond of Data

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GPS Applications
GPS Applications
Problem 1

GPS
Smart phone
Expensive watch
Fancy brick

1 day
4 hours
Problem I

1 day
4 hours

Start here

Expensive watch
Fancy brick
Phone

GPS
Problem II

t = 0: trigger → GPS starts

t = 1 ms

t = 30 s: position fix
Why are people still doing research on this topic?
Challenge 1

\[(t_{r,1} - t_{t,1}) \cdot c = d_1\]

\[(t_{r,2} - t_{t,2}) \cdot c = d_2\]

\[d_3 = (t_{r,3} - t_{t,3}) \cdot c\]

\[\|p_r - p_{s,i}\|_2 + \Delta t \cdot c = (t_{r,i} - t_{t,i}) \cdot c\]
GPS signals

Carrier

C/A code

GPS signal
Correlation

\[
\begin{align*}
\text{+1} & \quad \begin{array}{cccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array} = 13 \\
\text{-1} & \quad \begin{array}{cccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array} = 1
\end{align*}
\]
Timing & Decoding

- 50 bps
- Time stamp sent every 6 seconds
Sub-ms time sync

- We already have some time information
- C/A codes start at whole milliseconds
- $t_{\text{transmit}} = k \times 1 \text{ ms}$
Coarse-Time Navigation (CTN)

- 1 ms ↔ 300 km
- Known approximate position → whole ms time sync
Challenge II

20,000 km

Signal 1000x weaker than noise

Signal 30x weaker than noise
Noise

- 1 ms is little

Dead end?!
Sum over satellites

Signals from all satellites have to be aligned!

- Increased SNR
Hypotheses

- Assumed receiver state: \((x, y, z, t)\)
- Satellite ranges known → signal alignment known
Best hypothesis?

- Many possible positions!
- Number of hypotheses proportional to size of search space.

Infinitely many hypotheses?
Discretization of search space: 1D

- Discrete samples $\rightarrow$ discrete positioning resolution
Discretization of search space: 2D
**Off-by-one error**

- Oversampling:
  - triangular peak
- 8 Msps $\rightarrow$ $\sim$1/8 lower
  $\rightarrow$ - 0.6 dB
- Loss incurred only for some satellites
Challenge III

- We do not only have to search in the position domain...
- Time offset → Satellite position error
- \(10 \text{ km} \times 10 \text{ km} \times 1 \text{ km} \times 1 \text{ min}, 8 \text{ Msps} \rightarrow 2.8 \text{ billion hypotheses}\)
Exhaustive search

- Brute force
- Possible, but slow
- \( \sim 20 \text{ k evaluations / s} \)
- Parallelizable
Branch and bound

- Explore promising regions first
- Discard “bad” regions
- Runtime: a few seconds (single thread, good SNR)
Related Work

- Liu et al. “Energy Efficient GPS Sensing with Cloud Offloading” (SenSys‘12, Best Paper)
  - CTN, suffer from noise

- Collective Detection
  - Various papers: 1) slow or 2) not optimal
    - Mathematical analysis of the superior robustness of “direct positioning”
Accuracy: Average of the $k$ best hypotheses

Our method (3D, 1ms)  Liu et al. (2D, 2ms)
Accuracy: 3D vs. 2D

Our method vs. Liu et al. (2D, 2ms)
Accuracy: Average of \( k \) fixes

![Graph showing the fraction of localizations with error smaller than a given value vs. localization error in meters for different values of \( k \) ranging from 1 ms to 30 ms. Each line represents a different value of \( k \) with the following colors: 1 ms (blue), 2 ms (orange), 3 ms (yellow), 5 ms (purple), 10 ms (green), 20 ms (light blue), and 30 ms (red).]
Tracking

- Branch-and-bound for initial fix
- First fix results in small search space
- Brute force subsequently
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