



Principles of Distributed Computing

Exercise 8: Sample Solution

1 Multi-Valued Agreement

The following protocol implements asynchronous multivalued Byzantine agreement according to Definition 9.8 (relaxed for randomized protocols). It uses digital signatures and calls a (randomized) protocol for binary Byzantine agreement.

Algorithm 1 Multi-Valued Agreement

```
1: upon propose( $v$ ):
2: send the signed message ( $value, v$ ) to all servers
3:
4: upon receiving  $2t + 1$  messages ( $value, *$ ) with proper signatures:
5: let  $m$  be the value  $v$  that occurs most often in the received value messages
6: let  $\Pi$  be the set of received value messages
7: send the message ( $majority, m, \Pi$ ) to all servers
8:
9: upon receiving  $n - t$  messages ( $majority, *$ ) with valid proofs:
10: if all values  $m$  in the received majority messages are the same then
11:   let  $M$  be the majority value
12:   propose 1 for binary agreement
13: else
14:   propose 0 for binary agreement
15: end if
16:
17: upon deciding for  $b$  in binary agreement:
18: if  $b = 1$  then
19:   decide for  $M$ 
20: else
21:   decide for a default value
22: end if
```

The protocol satisfies the standard validity condition because if all honest servers propose the same value, all honest servers obtain a unique m , all valid *majority* messages contain m , and all honest servers propose 1 for binary agreement.

Agreement and termination follow from a standard argument and from the properties of the binary agreement protocol.

2 Strong Agreement

The standard validity condition of Binary Byzantine agreement requires a particular outcome only if *all* honest servers propose the same value; but the complement is that some honest server proposed the opposite value, hence any decision "makes sense" because some honest server proposed it.

Let D denote the agreement domain with m values and $H \subset D$ the set of values proposed by the honest servers. The values in H are called *valid*. Towards a contradiction, suppose that $n \leq (m + 1)t$ and $|H| = m - 1$. Let the set of all honest servers be partitioned into A and B such that $|A| \leq (m - 1)t$ and $|B| = t$, such that for every $v \in H$ there are at most t servers who propose v .

The adversary now causes all corrupted servers to follow the protocol with the invalid input $u \in D \setminus H$. The adversary isolates the servers in B by delaying all messages from servers in B . Then the servers in A must reach agreement together with the corrupted servers. But since the corrupted servers follow the protocol, they cannot be distinguished from honest servers and the protocol will decide on u with some non-negligible probability. Since u is not valid, this contradicts strong validity.

Reference: M. Fitzi and J. A. Garay. Efficient player-optimal protocols for strong and differential consensus. In *Proc. 22nd ACM Symposium on Principles of Distributed Computing (PODC)*, 2003.