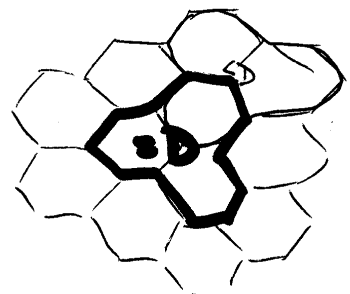
Greedy Algorithm is 3-competitive

Let σ be a sequence of calls.

Let D be the number of calls in any three mutually adjacent cells.

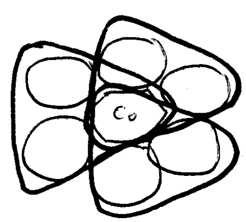
Then $cost_{OPT}(\sigma) \geq D$



Let c_0 be the cell that was assigned the highest frequency $\underline{a_0}$

Note $cost_{GREEDY}(\sigma) = a_0$ (by definition)

Let $N^*(c_0)$ be c_0 plus 6 neighboring cells



$$Calls(N^*(c_0)) \leq D + D + D - \underline{2 \cdot Calls(c_0)}$$

$$a_0 \leq 3D$$

$$\underline{cost_{GREEDY}(\sigma)} = a_0 \leq 3 \cdot D \leq \underline{3 \cdot cost_{OPT}(\sigma)}$$

$$\downarrow$$

$$\rho \leq 3$$

Remark

1.) cases how second highest etc stand to each other

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Rand. Call Control is 2,97 -competitive

σ: sequence of calls

$X(c)$: Prob that RANDOM accepted call c

$$E[\text{benefit}_{RANDOM}] = \sum_{c \in \sigma} X(c)$$

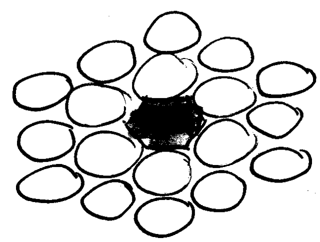
$$|A(\sigma)| = \text{benefit}_{OPT} = |\text{calls OPT accepted}|$$

Amortized benefit:

$$b(c) = X(c) + \sum_{c' \in N(c)} \frac{X(c')}{d(c')}, \quad \text{for each } c \in A(\sigma)$$

$\hookrightarrow d(c') = N(c) \cap A(c') \leq 3$

$$E[\text{benefit}_{RANDOM}] \stackrel{!}{=} \sum_{c \in A(\sigma)} b(c)$$



q = probability that no call was accepted in $N(c)$ when c is presented.

$$b(c) = X(c) + \sum_{c' \in N(c)} \frac{X(c')}{d(c')}$$

$$q \cdot p + (1-q) \cdot 0 + [q \cdot \geq 0] + (1-q)\,\frac{1}{3}$$

$\exists c^*$ in $N(c)$ mit $X(c^*) = 1$

$d(c^*) \leq 3$.

$$\Rightarrow b(c) \geq qp + (1-q)/3$$

$$q \geq (1-p)^6 \Rightarrow b(c) \geq (1-p)^6 (p - 1/3) + 1/3 \quad \underline{\text{max?}}$$

$$\frac{db(c)}{dp} = -6(1-p)^5(p - 1/3) + (1-p)^6 = \underbrace{(1-p)^5}_{>0} \underbrace{[-6p + 2 + 1 - p]}_{p = 3/7} \stackrel{!}{=} 0$$

$$\Rightarrow b(c) = \frac{8192}{2470629} + 1/3 = 0,33665$$

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Linearity of Expectations:

$$E[\text{benefit}_{RANDOM}] = \sum_{c \in A(\sigma)} b(c) = |A(\sigma)| \cdot 0{,}33665$$

$$\text{benefit}_{OPT} = |A(\sigma)|$$

$$\rho \cdot \text{benefit}_{RANDOM} \geq \text{benefit}_{OPT} \quad [\cancel{+c}]$$

$$\rho \geq \frac{1}{0{,}33665} \approx \underline{\underline{2.97}}$$

⚠ $q \geq (1-p)^6$ was bad approximation.

Use different p, depending on how many cells in $N(c)$ are marked

Ex: $N(c)$ marked = all 6 $\Rightarrow$ $p_6 = 1$