



# Principles of Distributed Computing

## Exercise 5

### 1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures (Chapter 5). Here, we will look at another prominent example, the *Pancake graph*  $P_n$ .

Define  $P_n$  as follows: the vertex set is

$$V(P_n) = \{v_1v_2 \dots v_n \mid v_i \in [n] \text{ and } v_i \neq v_j \forall i \neq j\} \quad (1)$$

where we use  $[n] = \{1, 2, \dots, n\}$ . In other words,  $V(P_n) = S_n$ , the group of all permutations on  $n$  elements. There exists an edge of dimension  $i$  for  $2 \leq i \leq n$  when

$$e_i = (u_1u_2 \dots u_i \dots u_n, v_1v_2 \dots v_i \dots v_n) \in E(P_n) \iff v_j = u_{i-j+1} \text{ for } 1 \leq j \leq i \text{ and } v_j = u_j \text{ for } i < j \leq n \quad (2)$$

or, we can say that an edge  $e_i$  represents a *prefix reversal*

$$v_1v_2 \dots v_iv_{i+1} \dots v_n \iff v_i \dots v_2v_1v_{i+1} \dots v_n. \quad (3)$$

For the following questions, where appropriate, give your answers in terms of  $N := |V(P_n)|$  (approximately), the number of vertices, as well as  $n$ .

- a) Draw (nicely!)  $P_n$  for  $n = 2, 3, 4$ . Try to describe a pattern for drawing  $P_n$  for any  $n$ .
- b) What is the degree of each vertex in  $P_n$ ?
- c) Can you give bounds on the diameter  $D(P_n)$  of the pancake network?
- d) (optional) Show that  $P_n$  is Hamiltonian for  $n \geq 3$ .

The pancake graph has recently been proposed for P2P networks, owing its usefulness to the above and other properties.