Principles of Distributed Computing
Exercise 5

1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures (Chapter 5). Here, we will look at another prominent example, the Pancake graph \( P_n \).

Define \( P_n \) as follows: the vertex set is

\[
V(P_n) = \{v_1v_2\ldots v_n | v_i \in [n] \text{ and } v_i \neq v_j \forall i \neq j \}
\]  

(1)

where we use \([n] = \{1, 2, \ldots, n\}\). In other words, \( V(P_n) = S_n \), the group of all permutations on \( n \) elements. There exists an edge of dimension \( i \) for \( 2 \leq i \leq n \) when

\[
e_i = (u_1u_2\ldots u_i\ldots u_n, v_1v_2\ldots v_i\ldots v_n) \in E(P_n) \iff v_j = u_{i-j+1} \text{ for } 1 \leq j \leq i \text{ and } v_j = u_j \text{ for } i < j \leq n
\]  

(2)

or, we can say that an edge \( e_i \) represents a prefix reversal

\[
v_1v_2\ldots v_i v_{i+1} \ldots v_n \leftrightarrow v_1 \ldots v_2 v_1 v_{i+1} \ldots v_n.
\]  

(3)

For the following questions, where appropriate, give your answers in terms of \( N := |V(P_n)| \) (approximately), the number of vertices, as well as \( n \).

a) Draw (nicely!) \( P_n \) for \( n = 2, 3, 4 \). Try to describe a pattern for drawing \( P_n \) for any \( n \).

b) What is the degree of each vertex in \( P_n \)?

c) Can you give bounds on the diameter \( D(P_n) \) of the pancake network?

d) (optional) Show that \( P_n \) is Hamiltonian for \( n \geq 3 \).

The pancake graph has recently been proposed for P2P networks, owing its usefulness to the above and other properties.