Principles of Distributed Computing  
Exercise 2: Sample Solution

1. Leader Election in an “almost anonymous” Ring

   a) Yes, it is possible:

   **Algorithm 1** Leader Election (all but one nodes have the same ID)

   1: send IDs two hops around the ring
   2: if the same ID has been received twice AND the received ID differs from the own ID then
   3: I am the leader
   4: end if

2. Distributed Computation of the AND

   a) Because the size of the ring is not known to the nodes, the case where all nodes have a one as input and the case where all but one nodes have a one as input cannot be distinguished.

   b) All input values have to be sent all around the ring. In order to detect the returning of the own message, we add a hop counter to each message. If the message has made \( n \) hops, it has arrived where it started.

   c) The following algorithm calculates the AND in a synchronous, non-uniform ring:

   **Algorithm 2** AND in the Ring: asynchronous, non-uniform (\( n \) is the number of nodes)

   1: if input bit = 0 then
   2: send 0 to the neighbor in the ring
   3: end if;
   4: for \( i := 2 \) to \( n \) do
   5: if received a 0 and have not already sent a 0 then
   6: send 0 to the neighbor in the ring
   7: end if
   8: end for;
   9: if received at least one 0 then
   10: result := 0
   11: else
   12: result := 1
   13: end if;

   If the result is 1, no message is sent, otherwise there is exactly one message over each link. Thus, time and message complexity are both \( n \).