1 Task 1

Consider the network failure problem for the special case of Tree Topology. That is, our network has the tree structure; however, it is not necessarily a binary tree. Your goal is to find a detection set that can detect all \((\epsilon, k)\) edge partitions; that is, for any network partition caused by removing at most \(k\) edges, if there are two components, each of size at least \(\epsilon n\), then your detection set must have a vertex (detector) in each component.

Show that for the tree network, there is always a detection set with \(O\left(\frac{k}{\epsilon}\right)\) vertices. Give an algorithm for finding such a set; analyze its time complexity; and give a proof of correctness that your chosen set of \(O\left(\frac{k}{\epsilon}\right)\) vertices do form a detection set.

2 Task 2

Consider a slightly modified version of detection set than the one considered by Kleinberg. The set of elements \(Z\) destroyed by the adversary can include both edges and vertices.

A detection set \(D\) is called a Weak Detection Set if, for every \((\epsilon, k)\) partitioning set \(Z\),

- either \(Z\) intersects \(D\) (adversary kills a detection node), or
- there are two nodes that lie in different components of \(G \setminus Z\) (who cannot communicate, and hence act as witnesses).

Notice that this is different from Kleinberg’s model, because Kleinberg’s model allows the possibility that an adversary-captured node can become malicious and may not be detectable. Show that for this model, we can always find an \((\epsilon, k)\) detection set of size \(O\left(\frac{k}{\epsilon}\right)\).