Principles of Distributed Computing

Exercise 7

1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures (Chapter 8). Here, we will look at another prominent example, the Pancake graph $P_n$.

Define $P_n$ as follows: the vertex set is

$$V(P_n) = \{v_1v_2\ldots v_n \mid v_i \in [n] \text{ and } v_i \neq v_j \forall i \neq j\}$$

(1)

where we use $[n] = \{1, 2, \ldots, n\}$. In other words, $V(P_n) = S_n$, the group of all permutations on $n$ elements. There exists an edge of dimension $i$ for $2 \leq i \leq n$ when

$$e_i = (u_1u_2\ldots u_n, v_1v_2\ldots v_i\ldots v_n) \in E(P_n) \iff v_j = u_{i-j+1} \text{ for } 1 \leq j \leq i \text{ and } v_j = u_j \text{ for } i < j \leq n$$

(2)

or, we can say that an edge $e_i$ represents a prefix reversal

$$v_1v_2\ldots v_iv_{i+1}\ldots v_n \iff v_1\ldots v_2v_1v_{i+1}\ldots v_n.$$  

(3)

For the following questions, where appropriate, give your answers in terms of $N := |V(P_n)|$ (approximately), the number of vertices, as well as $n$.

a) Draw (nicely!) $P_n$ for $n = 2, 3, 4$. Try to describe a pattern for drawing $P_n$ for any $n$.

b) What is the degree of each vertex in $P_n$?

c) Can you give bounds on the diameter $D(P_n)$ of the pancake network?

d) (optional) Show that $P_n$ is Hamiltonian for $n \geq 3$.

The pancake graph has recently been proposed for P2P networks, owing its usefulness to the above and other properties.