Greedy Algorithm on Graph

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Based on the papers:

- S.Davis, R.Impagliazzo "Models of greedy Algorithms for Graph Problems"

- A.Borodin, J.Boyar, K.S.Larsen "Priority Algorithms for Graph Optimization Problems"

Introduction

Greedy Algorithms are a typology of Algorithms of substantial importance. Beside the simplicity of the underlying principle, which basically is to make the choice that according with a determined parameter will produce the most favorable situation in the next step, there is an emotional component that makes them appealing because of the (sometime debatable) common feeling that these kind of algorithm easy to grasp even for the non experts. Difficulties arise while trying to find a rigorous definition embracing all the algorithm that in almost forty years have been defines within the rather unsharp boundarys of this definition.
Instead of venture on the slippery ground of finding the ultimate definition of greedy algorithm, that beside mantaining a semantical bound with the concept of greed should at the same time be precise enough to allow to mathematically deal with it and stay broad enough to comprise the up to now defined algorithms, the authors of the papers decided to take another approach and explored the properties of a model of algorithm, the so called priority model, that is based on a rigourous definition and whom most of the greedy algorithm can be lead back to.

Priority Model

The idea underlying the Priority Model belonging algorithms is that at each step the not yet treated elements are classified according to an ordering, something is executed on the first element of the ordering, the execution is committed and the algorithm can't no longer act on this element.
The Priority Model basically defines two class of algorithms, the Fixed Priority and the Adaptive Priority class.
Fixed Priority algorithm are so defined that the ordering is determined at the very beginning of the execution of the algorithm and afterwards is not possible to change it; on the contrary Adaptive Priority algorithms redefine the ordering at every step, eliminating the already treated elements and

using information on the already treated elements to define a better ordering.
What makes algorithms of both Priority classes "greed-alike" is that they look at the yet to be treated element as if these were uncorrelated entities.
Two subclasses of the Adaptive Priority are also introduced for problem resulting in accept/reject of elements: Memoryless and Acceptance-First.
Memoryless Algorithm Class represent a restriction of Adaptive Priority Class in that the decision about an element can't take into account the previously rejected elements (which are in a certain sense forgotten); Acceptance-First algorithms are so defined that after the first reject occurs no further accept are allowed.

The Results

The papers basically use the Priority Model to show lower bounds of greed algorithms applied to a vast subset of the most common graph problems.
The inclusion of the Fixed Priority Class into the Adaptive Priority Class (rather foreseeable indeed) and the equivalence under certain conditions of the Acceptance-First and Memoryless algorithms are possibly the only results that are not lower bounds
Even tough some results are actually interesting (for instance the lower bound for the Vertex Coloring Problem which shows that it we already know algorithms with the best possible lower bound) many of the founded lower bound are of no special meaning.
Moreover most of the theorems hold only if the graph problem is represented using either the edge model or the vertex model representation, which seems to be due to some technicalities in the proofs.
The Memoryless and Acceptance-First classes result somewhat useful in many proofs, unfortunately they define object that per se have no special meaning and it looks even like they have been introduced because of mere (mathematic) technical reasons

Conclusion

The model is interesting and in line of principle quite promising especially as it allows to proove assumptions about graph yet to be designed, nevertheless it seems that up to now this proceeding has not yet led to a breakthrough; furthermore even tough almost all the proofs are based on the competitive analysis (greedy algorithm are in many way online-alike) they are all quite different, i.e. it seems there is not yet an "almost standard" approach that let to demonstrate wide ranges of properties.