ETH
Eidgenössische Technische Hochschule Zürich
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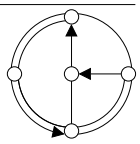

WS 2005/06 Prof. R. Wattenhofer / Thomas Moscibroda / Stefan Schmid

Discrete Event Systems
Exercise 6

1 Chomsky Normal Form

Convert the following CFGs into an equivalent CFGs in *Chomsky normal form*.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow 00 \mid \varepsilon. \end{aligned}$$

2 CFL Closure Properties

Prove that the class of CFLs is closed under the regular operators union, catenation, and Kleene star. Formally, let L and L' be languages generated by the context free grammars G and G', respectively. Show that the languages $L \cup L'$, LL', and L^* are context free languages.

3 Context Sensitive Languages

In this exercise you will study a sample language which is not context-free: $L = \{zz | z \in \{0,1\}^*\}$.

a) Prove that L is not context-free.
Hint: Tandem pumping!

b) Show that L is context-sensitive by providing a corresponding grammar.

4 Transducer-Robot

The goal of this exercise is to program a robot to follow a wall. This is an example for a *transducer*. See Figure 1. The robot's world is a grid, any square is either free or occupied by a wall. The robot, shown as an arrow, is placed on an arbitrary free square facing one of the four possible directions. The robot has two binary sensors (inputs): **h** (head) signals whether the square in front of the robot is free ($h = 0$) or occupied by a wall ($h = 1$). **r** (right) provides the same information for the square to the right of the robot. The robot is capable of two primitive actions: **R** turns right by 90° while remaining on the same square; **F** (forward) advances one square in the robot's current direction. Note that the robot's actions correspond to an output alphabet, hence the robot is a *transducer*.

The robot must be programmed to find a piece of wall, then endlessly cycle along the inside of the wall, touching it with its right side!

Program the robot as a transducer that solves this task!

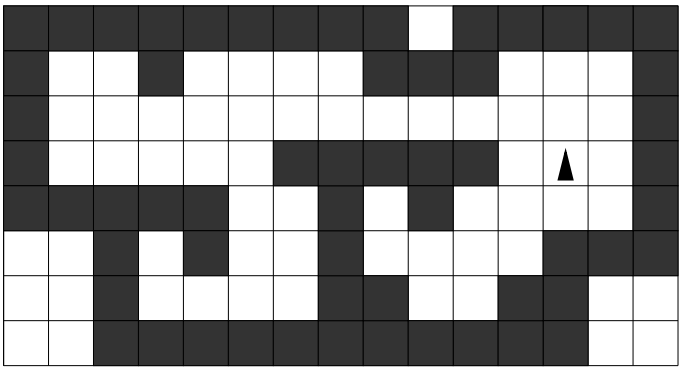

Figure 1: The robot's world is limited!