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Discrete Event Systems
Exercise 10: Sample Solution

1 Gloriabar

a) The situation can be modeled by a M/M/1 queue. We have an arrival rate of $\lambda = 540/(90 \cdot 60) = 0.1$ (persons per second), and $\mu = 1/9$ (persons per second). Thus $\rho = \lambda/\mu = 0.9$. Therefore, the expected waiting time is $W = \rho/(\mu - \lambda) = 81$ seconds. The expected time until the student gets her menu is given by $T = 1/(\mu - \lambda) = 90$ seconds.

b) The queue length is given by $N = \rho^2/(1 - \rho) = 8.1$.

c) We require that $T = 1/(\mu - 0.1) = 90/2$. Thus, $\mu = 11/90$, i.e., instead of 9 secs, the service time should be roughly $90/11 = 8.2$ secs.

2 Queuing Networks

a) See Figure 1.

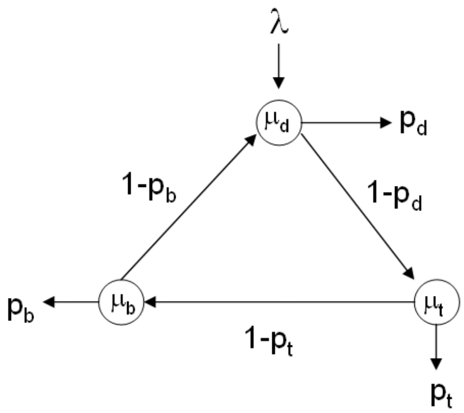

Figure 1: Queuing Network.

b) We have an open queuing network an hence we can apply Jackson's theorem (slides 97ff):

$$\lambda_d = \lambda + \lambda_b(1 - p_b) \quad (1)$$
$$\lambda_t = \lambda_d(1 - p_d) \quad (2)$$
$$\lambda_b = \lambda_t(1 - p_t) \quad (3)$$

Solving this equation system gives:

$$\lambda_d = \frac{\lambda}{1-(1-p_d)(1-p_t)(1-p_b)}$$
$$\lambda_t = \frac{(1-p_d)\lambda}{1-(1-p_d)(1-p_t)(1-p_b)}$$
$$\lambda_b = \frac{(1-p_d)(1-p_t)\lambda}{1-(1-p_d)(1-p_t)(1-p_b)}$$

c) The waiting time is given by $W_t = \rho_t/(\mu_t - \lambda_t)$, where $\rho_t = \lambda_t/\mu_t$.

d) We have

$$\lambda_d = 10, \quad \lambda_t = 25/3, \quad \lambda_b = 20/3$$
$$\rho_d = 1/2, \quad \rho_t = 5/6, \quad \rho_b = 2/3.$$

Therefore, by the formula of slide 79, the number of customers in the system is given by

$$N = \frac{\lambda_d}{\mu_d - \lambda_d} + \frac{\lambda_t}{\mu_t - \lambda_t} + \frac{\lambda_b}{\mu_b - \lambda_b} = 8.$$

Applying Little's formula to the entire system gives $T = N/\lambda = 8/5$ hours.

e) We have

$$\lambda_t = \frac{(1-p_d)\lambda}{1-(1-p_d)(1-p_t)(1-p_b)} = 1 \Leftrightarrow p_d = 23/28.$$

3 Theory of Ice Cream Vending

The situation can be described by a classic M/M/2 system. According to slide 90, there is an equilibrium iff

$$\rho = \lambda/(2\mu) < 1.$$

For the stationary distribution, in holds that

$$\pi_0 = \frac{1}{1+2\rho+4\rho^2/(2(1-\rho))} = \frac{1-\rho}{1+\rho}.$$