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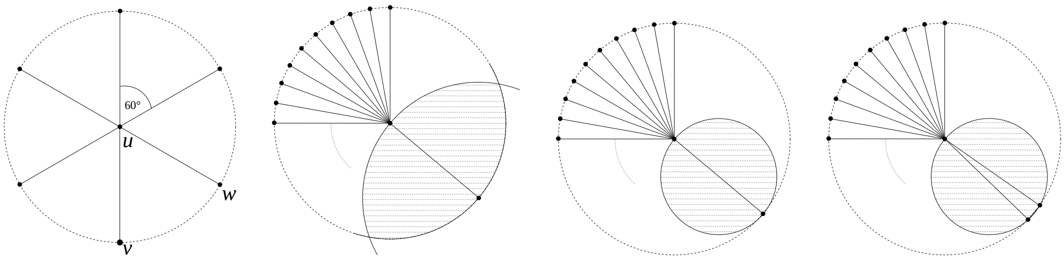
Mobile Computing
Exercise 6, Sample Solution

1 Degree of Euclidean Graphs

From the four considered algorithms the Minimum Spanning Tree (MST) is the only one resulting in a bounded-degree topology. Figure 1 depicts worst-case instances for the different graphs. Figure 1(a) gives thereby an idea why the degree of the MST is bounded to six. If the angle between two adjacent edges (u, v) and (u, w) is less than 60 degrees it would be cheaper to abandon one of these edges and add (v, w) to the final topology. By the definition of the Relative Neighborhood Graph (RNG), an edge is discarded if a node is situated inside the lune of this edge (see Figure 1(b)). However, the boundary is not included. If we therefore arrange n nodes on a circle around another node, we end up with a maximum degree of n. The same worst-case example also exists for the Gabriel Graph (GG) and the Delaunay Triangulation (DT) (cf. Figure 1(c) and (d)). With these topologies the critical areas depicted in Figure 1 do not contain any other node on the circle around the center node. Thus, we also obtain a graph with unbounded degree in the worst case.



(a) MST (b) RNG (c) GG (d) DT

Figure 1: Bad network instances for the four Euclidean graphs. One can see that only the MST has bounded degree.

2 Gabriel Graph Spanner Property

It can be shown that the Gabriel Graph (GG) is no Euclidean spanner but has a spanner ratio in $\Omega(\sqrt{n})$. We do not show a detailed proof here but rather give a proof sketch. In Figure 2 a bad network instance is depicted. In order to clarify the example we assume that the construction of the GG is done in steps. We first consider the edge (u, v). We see that this edge is replaced by a detour via an additional node (indicated by the two arrows labeled with 1). However these edges are also replaced in a second step. If we develop this recursive concept, one can see that we obtain a fractal like construction. Using the Pythagorean theorem we end up with a path from u to v with length in $O(\sqrt{n})$.

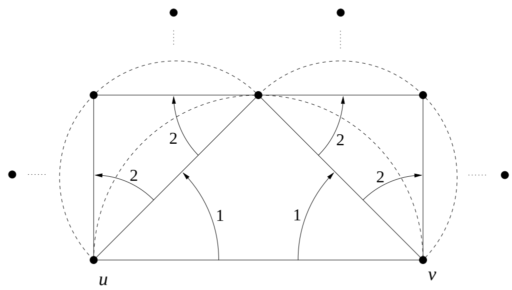


Figure 2: Fractal construction of a worst-case instance for the Gabriel Graph.

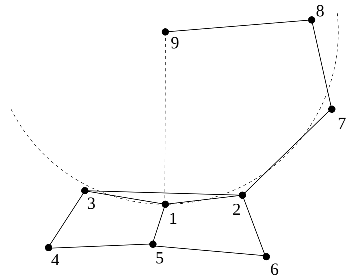


Figure 3: Bad instance for "closest-node" face routing.

3 Geographic Routing

If we apply the "closest-node" rule we cannot guarantee message delivery anymore. Figure 3 shows a counterexample. Let's assume that node 1 wants to send a message to node 9. First, it would route the message along the face (1,2,3) in order discover that there are no other nodes closer to 9 than itself. Then it switches the face (either (1,3,4,5) or (1,5,6,2)). However, none of the other nodes on these faces is closer to 9 and the message would end up in being sent along these three faces forever.

4 Non-Planar Graphs

Face routing is dependent on planar graphs. Consider the example graph in Figure 4. In this topology consisting of six nodes we would like to route a message from node s to node t. Node s therefore sends the message along the face that is intersected by the line from s to t. The path of the message is thereby indicated by arrows in Figure 4. However, as the message arrives at v we are on the wrong side of the face to catch the exit path to node t. Once the message has traversed the whole face and returned to s it has not found a point closer to the target t than s. Thus, the message gets stuck at s.

In the Quasi Unit Disk Graph model we are able to apply face routing if the parameter d is greater or equals to $1/\sqrt{2}$. The reason for this is that we are able to detect all intersections in the graph locally. Let $e = (u, v)$ be an edge and w be a node which is in the disk with diameter (u, v). Either u and w or v and w are connected by an edge. This is illustrated in Figure 5. Furthermore, it holds that intersecting edges are connected directly via a third edge. This follows from a simple angle argument. Using these two properties we are able to extend our graph with virtual edges. For example the edge (v, w) in Figure 5 might not exist but can be replaced by a virtual edge. Whenever a packet should be sent over (v, w) it is relayed via u. Once all these edges are added to the graph we can thus compute the Gabriel Graph as usual resulting in a planar topology. Thus face routing still works of $d \geq 1/\sqrt{2}$.

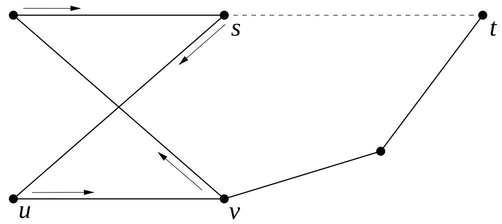


Figure 4: Counterexample for face routing on non-planar graphs.

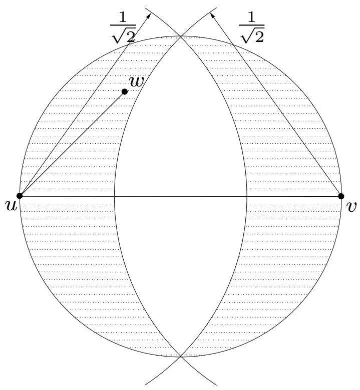


Figure 5: w is either connected to u or to v.