Chapter 7
TOPOLOGY CONTROL

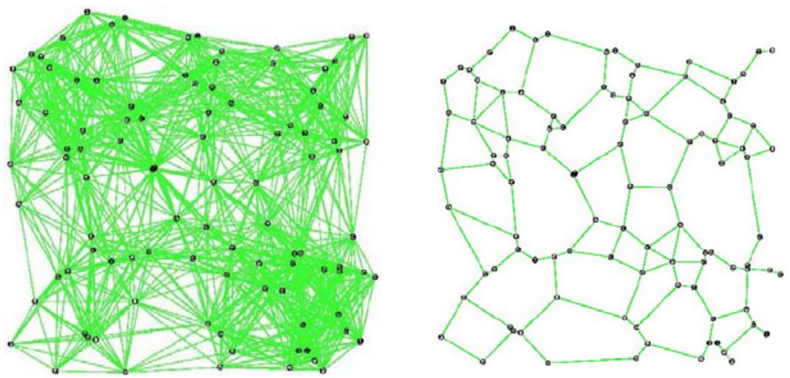
Distributed Computing Group

Mobile Computing
Winter 2005 / 2006

Overview – Topology Control

- Gabriel Graph et al.
- XTC
- Interference
- SINR & Scheduling Complexity

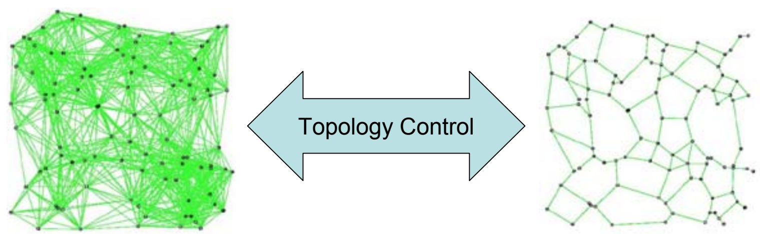
Topology Control



- Drop long-range neighbors: Reduces interference and energy!
- But still stay connected (or even spanner)

Topology Control as a Trade-Off

Sometimes also clustering, Dominating Set construction (See later)



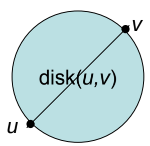
Network Connectivity
Spanner Property

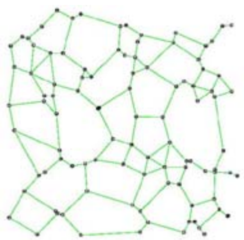
$d(u,v) \cdot t \geq d_{TC}(u,v)$

Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

Gabriel Graph

- Let disk(*u*,*v*) be a disk with diameter (*u*,*v*) that is determined by the two points *u*,*v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the disk(*u*,*v*) including boundary contains no other points.

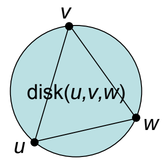


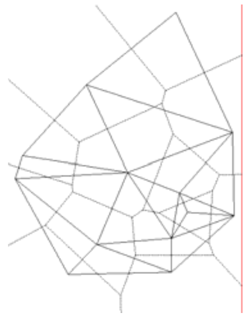


- As we will see the Gabriel Graph has interesting properties.

Delaunay Triangulation

- Let disk(*u*,*v*,*w*) be a disk defined by the three points *u*,*v*,*w*.
- The Delaunay Triangulation (Graph) DT(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is a triangle of edges between three nodes *u*,*v*,*w* iff the disk(*u*,*v*,*w*) contains no other points.

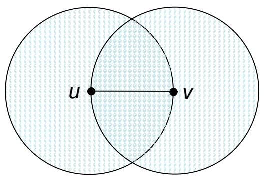




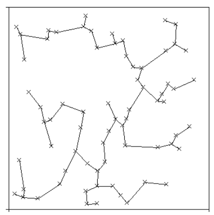
- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas; the DT is planar; the distance of a path (s,…,t) on the DT is within a constant factor of the s-t distance.

Other planar graphs

- Relative Neighborhood Graph RNG(V)
- An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)
- A subset of *E* of *G* of minimum weight which forms a tree on *V*.

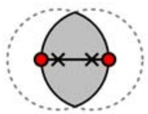


Properties of planar graphs

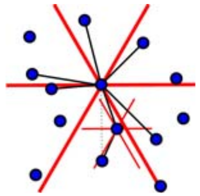
- Theorem 1:
  $MST(V) \subseteq RNG(V) \subseteq GG(V) \subseteq DT(V)$
- Corollary:
  Since the MST(V) is connected and the DT(V) is planar, all the planar graphs in Theorem 1 are connected and planar.
- Theorem 2:
  The Gabriel Graph contains the Minimum Energy Path (for any path loss exponent $\alpha \geq 2$)
- Corollary:
  GG(V) ∩ UDG(V) contains the Minimum Energy Path in UDG(V)

More examples

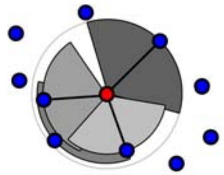
- β-Skeleton
  - Generalizing Gabriel (β = 1) and Relative Neighborhood (β = 2) Graph



- Yao-Graph
  - Each node partitions directions in k cones and then connects to the closest node in each cone



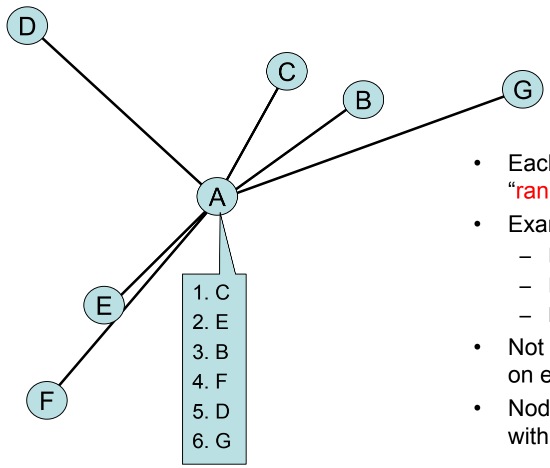
- Cone-Based Graph
  - Dynamic version of the Yao Graph. Neighbors are visited in order of their distance, and used only if they cover not yet covered angle



XTC: Lightweight Topology Control

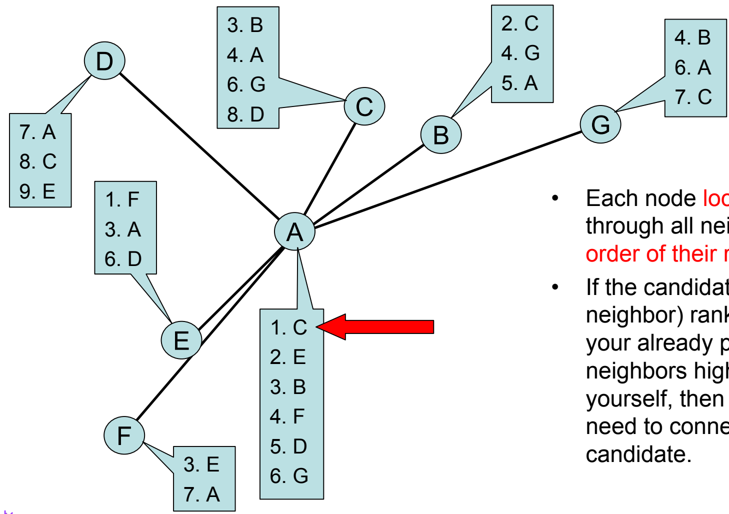
- Topology Control commonly assumes that the node positions are known.
- What if we do not have access to position information?
- XTC algorithm
- XTC analysis
  - Worst case
  - Average case

XTC: lightweight topology control without geometry



- Each node produces "ranking" of neighbors.
- Examples
  - Distance (closest)
  - Energy (lowest)
  - Link quality (best)
- Not necessarily depending on explicit positions
- Nodes exchange rankings with neighbors

XTC Algorithm (Part 2)



- Each node locally goes through all neighbors in order of their ranking
- If the candidate (current neighbor) ranks any of your already processed neighbors higher than yourself, then you do not need to connect to the candidate.

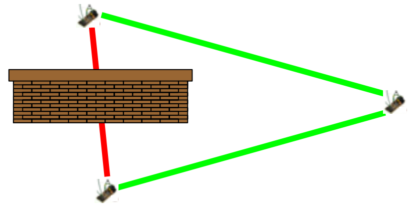
XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.
- Proof:
  - Assume 1) $u \rightarrow v$ and 2) $u \not\leftarrow v$
  - Assumption 2) $\Rightarrow \exists w$: (i) $w \prec_v u$ and (ii) $w \prec_u v$

Contradicts Assumption 1)

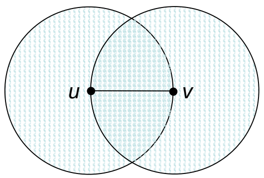
XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.
- Connectivity: If two nodes are connected originally, they will stay so (provided that rankings are based on symmetric link-weights).
- If the ranking is energy or link quality based, then XTC will choose a topology that routes around walls and obstacles.

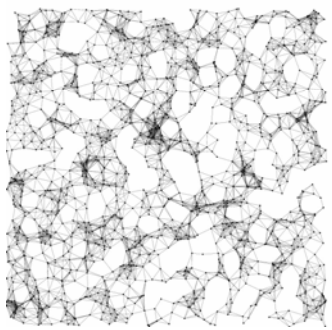


XTC Analysis (Part 2)

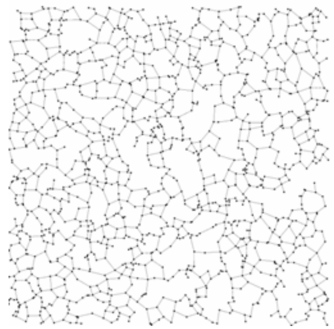
- If the given graph is a Unit Disk Graph (no obstacles, nodes homogeneous, but not necessarily uniformly distributed), then …
- The degree of each node is at most 6.
- The topology is planar.
- The graph is a subgraph of the RNG.
- Relative Neighborhood Graph RNG(V):
- An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



XTC Average-Case

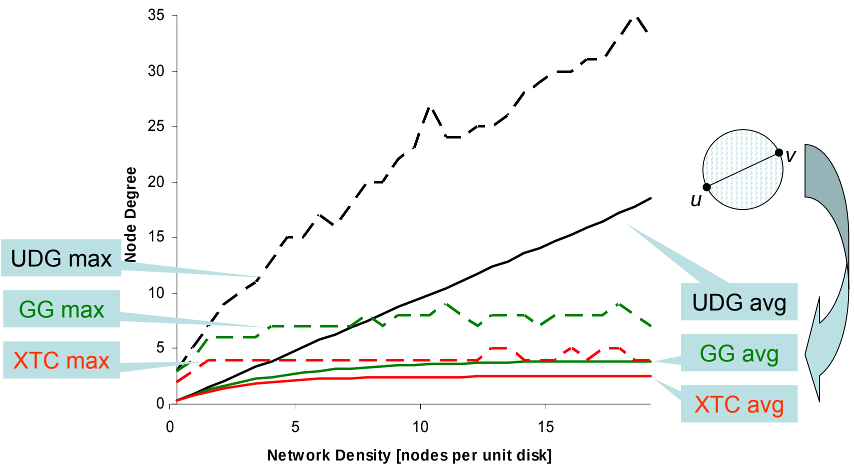


Unit Disk Graph

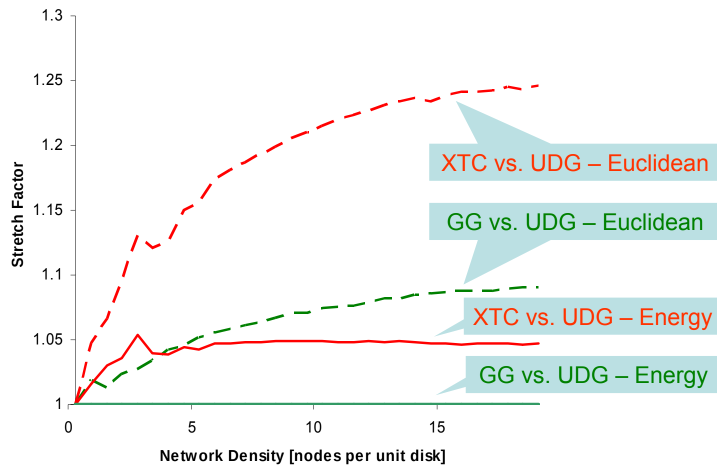


XTC

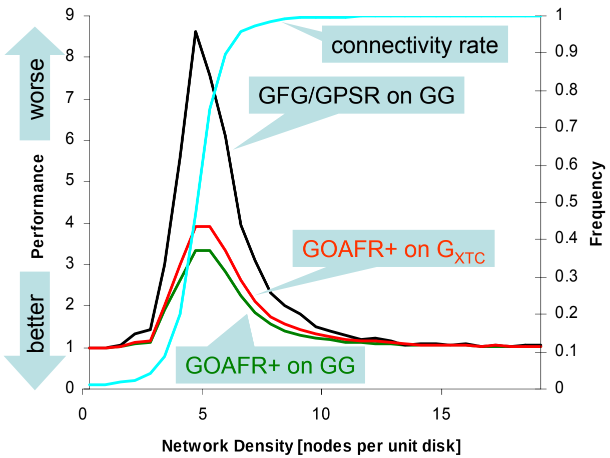
XTC Average-Case (Degrees)



XTC Average-Case (Stretch Factor)



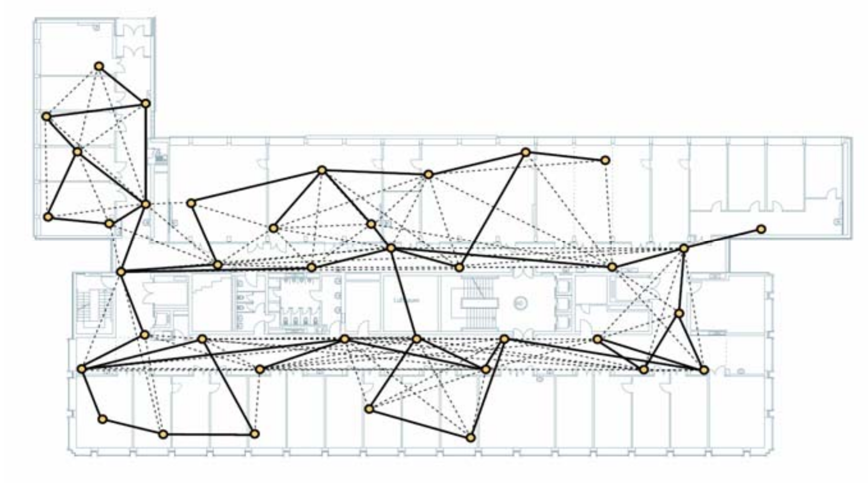
XTC Average-Case (Geometric Routing)



k-XTC: More connectivity

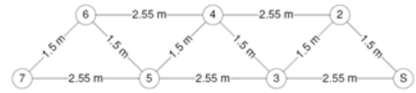
- A graph is k-(node)-connected, if k-1 arbitrary nodes can be removed, and the graph is still connected.
- In k-XTC, an edge (u,v) is only removed if there exist k nodes w_1, …, w_k such that the 2k edges (w_1, u), …, (w_k, u), (w_1,v), …, (w_k,v) are all better than the original edge (u,v).
- Theorem: If the original graph is k-connected, then the pruned graph produced by k-XTC is as well.
- Proof: Let (u,v) be the best edge that was removed by k-XTC. Using the construction of k-XTC, there is at least one common neighbor w that survives the slaughter of k-1 nodes. By induction assume that this is true for the j best edges. By the same argument as for the best edge, also the j+1st edge (u',v'), since at least one neighbor survives w' survives and the edges (u',w') and (v',w') are better.

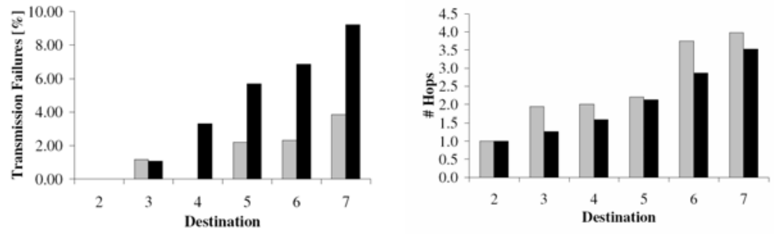
Implementing XTC, e.g. BTnodes v3



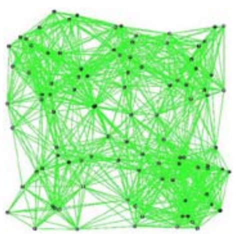
Implementing XTC, e.g. on mica2 motes

- Idea:
  - XTC chooses the reliable links
  - The quality measure is a moving average of the received packet ratio
  - Source routing: route discovery (flooding) over these reliable links only

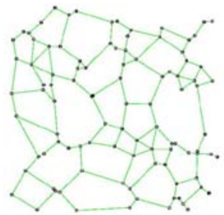




Topology Control as a Trade-Off



Topology Control



Network Connectivity
Spanner Property

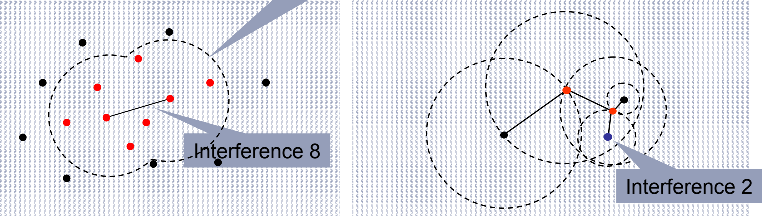
Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

Really?!?

What is Interference?

Exact size of interference range does not change the results

Link-based Interference Model | Node-based Interference Model



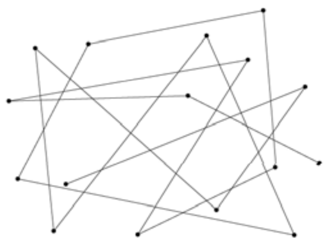
„How many nodes are affected by communication over a given link?“

„By how many other nodes can a given network node be disturbed?“

- Problem statement
  - We want to minimize maximum interference
  - At the same time topology must be connected or a spanner etc.

Low Node Degree Topology Control?

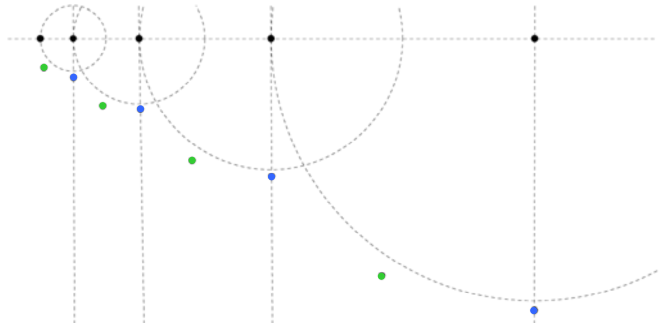
Low node degree does not necessarily imply low interference:



Very low node degree but huge interference

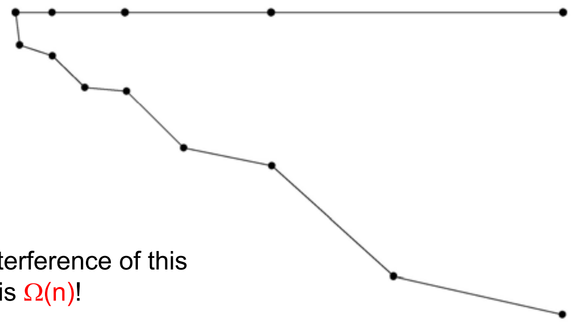
Let's Study the Following Topology!

…from a worst-case perspective



Topology Control Algorithms Produce…

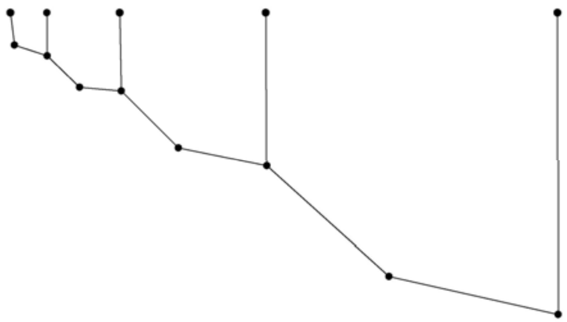
- All known topology control algorithms (with symmetric edges) include the nearest neighbor forest as a subgraph and produce something like this:



- The interference of this graph is $\Omega(n)$!

But Interference…

- Interference does not need to be high…



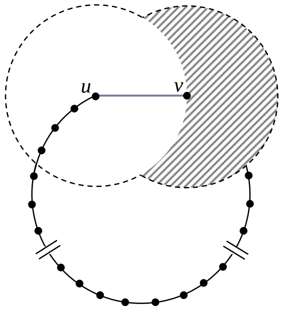
- This topology has interference $O(1)$!!

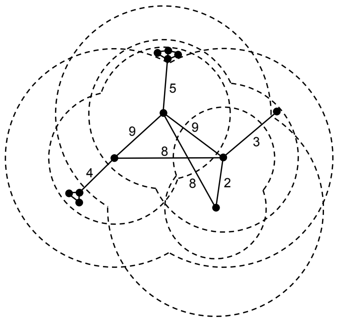
Link-based Interference Model

- Interference-optimal topologies:

There is no local algorithm that can find a good interference topology

The optimal topology will not be planar





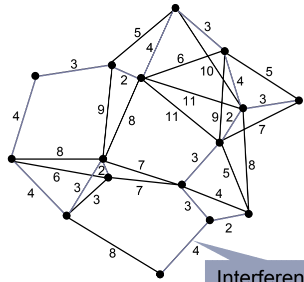
Link-based Interference Model

- LIFE (Low Interference Forest Establisher)
  - Preserves Graph Connectivity

LIFE

- Attribute interference values as weights to edges
- Compute minimum spanning tree/forest (Kruskal's algorithm)

LIFE constructs a minimum-interference forest



Interference 4

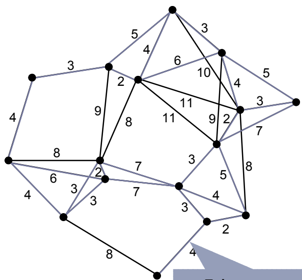
Link-based Interference Model

- LISE (Low Interference Spanner Establisher)
  - Constructs a spanning subgraph

LISE

- Add edges with increasing interference until spanner property fulfilled

LISE constructs a minimum-interference t-spanner



5-hop spanner with Interference 7

Link-based Interference Model

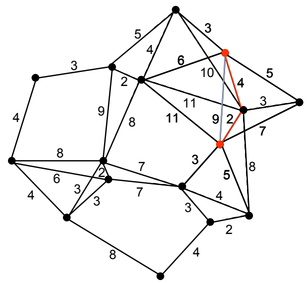
- LocaLISE
  - Constructs a spanner locally

Scalability

LocaLISE

- Nodes collect (t/2)-neighborhood
- Locally compute interference-minimal paths guaranteeing spanner property
- Only request that path to stay in the resulting topology

LocaLISE constructs a minimum-interference t-spanner



Link-based Interference Model

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LocaLISE

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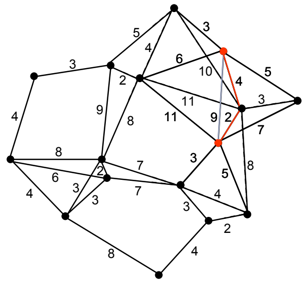
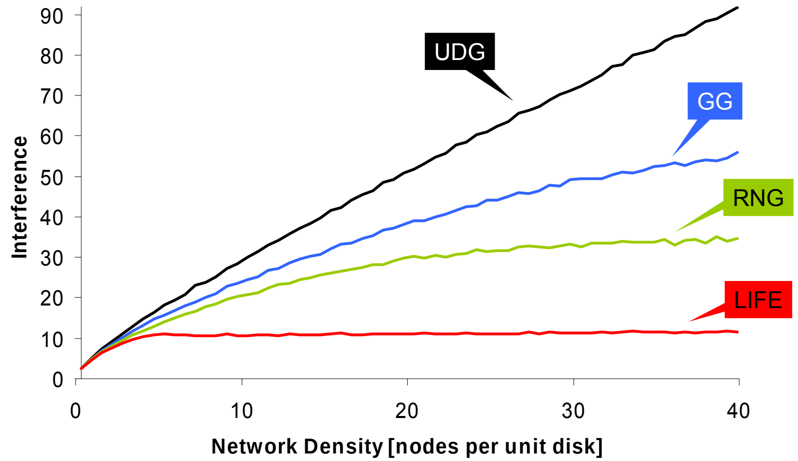
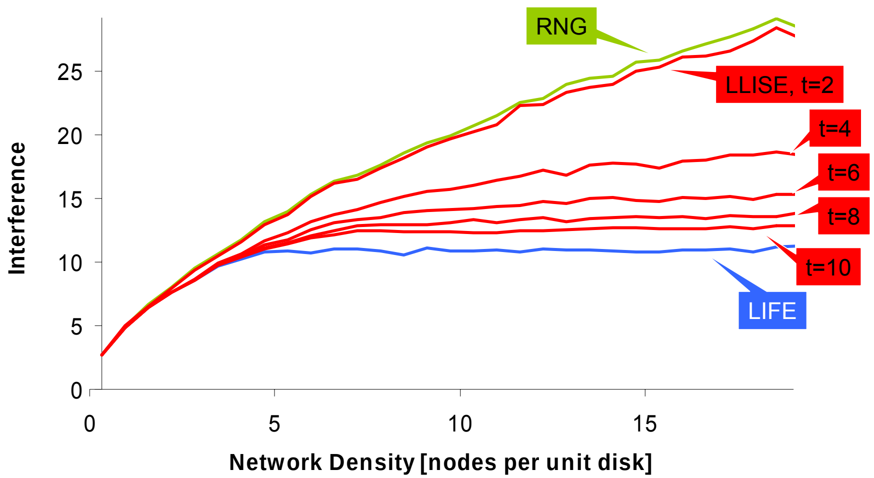


LocaLISE constructs a minimum-interference t-spanner

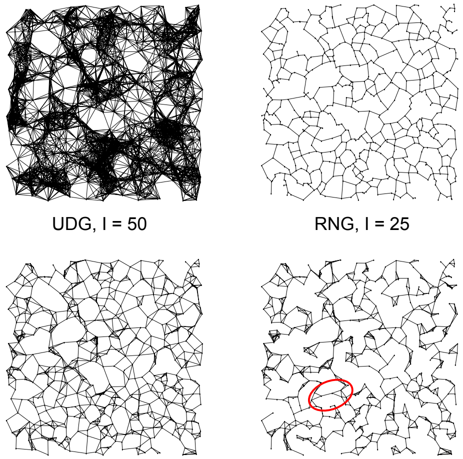
Average-Case Interference: Preserve Connectivity



Average-Case Interference: Spanners



Link-based Interference Model

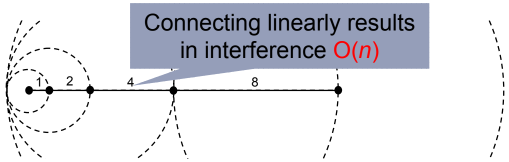


UDG, I = 50 | RNG, I = 25

LocaLISE$_2$, I = 23 | LocaLISE$_{10}$, I = 12

Node-based Interference Model

- Already 1-dimensional node distributions seem to yield inherently high interference...



Connecting linearly results in interference O(*n*)

- ...but the exponential node chain can be connected in a better way

Node-based Interference Model

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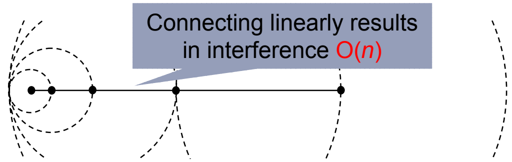


Connecting linearly results in interference O(*n*)

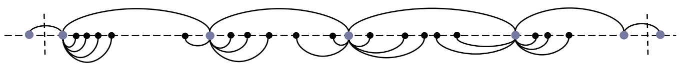
- ...but the exponential node chain can be connected in a better way

⇒ Interference $\in O(\sqrt{n})$

Matches an existing lower bound

Node-based Interference Model

- Arbitrary distributed nodes in one dimension
  - Approximation algorithm with approximation ratio in O($\sqrt[4]{n}$)



- Two-dimensional node distributions
  - Randomized algorithm resulting in interference O($\sqrt{n \log n}$)
  - No deterministic algorithm so far...

Towards a More Realistic Interference Model...

- Signal-to-interference and noise ratio (SINR)

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N + \sum_{w \in V \setminus \{u\}} \frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

Power level of node u

Path-loss exponent

Noise

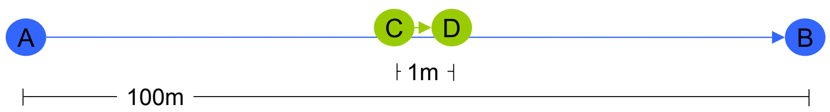
Minimum signal-to-interference ratio

Distance between two nodes

- Problem statement
  - Determine a power assignment and a schedule for each node such that all message transmissions are successful

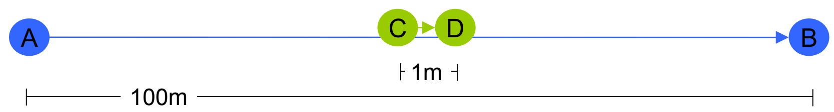
SINR is always assured

Quiz: Can these two links transmit simultaneously?



- Graph-theoretical models: No!
  - Neither in- nor out-interference
- SINR model: constant power: No!
  - Node B will receive the transmission of node C
- SINR model: power according to distance-squared: No!
  - Node D will receive the transmission of node A

Let's try harder…



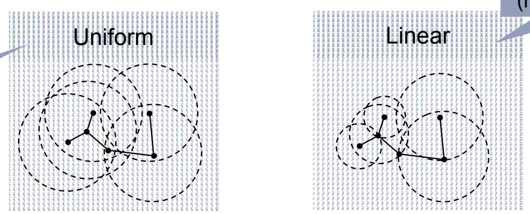
A Simple Problem

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N+\sum_{w\in V\setminus\{u\}}\frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

- Each node in the network wants to send a message to an arbitrary other node
  - Commonly assumed power assignment schemes

Proportional to (receiver distance)$^\alpha$

Constant power level



➡ Both lead to a schedule of length $\in \Theta(n)$

  - A clever power assignment results in a schedule of length $\in \boldsymbol{O(\log^3 n)}$

This has strong implications to MAC layer protocols