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Distributed Computing Group

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Discrete Event Systems
Exercise 4

1 Regular and Context-Free Languages

- Consider the following context-free grammar $G : S \rightarrow SS|1S2|0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.
- The regular languages are a subset of the context-free languages. Give the context-free grammar for a language L that is regular.

2 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

- $L = \{w|$ the length of w is odd$\}$
- $L = \{w|$ contains more 1s than 0s$\}$

3 Pushdown Automata

Consider the following context-free grammar G with non-terminals S and A, start symbol S, terminals '(', ')', and '0':

$$\begin{array}{rcl} S & \rightarrow & SA \,|\, \epsilon \\ A & \rightarrow & (S) \,|\, 0 \end{array}$$

a) What are the 4 shortest strings produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

4 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C, i.e., a register than can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M. M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let L_{count} be the set of languages recognized by counter automata.

- Let L_{reg} be the set of regular languages. Prove that $L_{reg} \in L_{count}$.
- Prove that the opposite is not true, that is, $L_{count} \notin L_{reg}$. Do so by giving a language which is in L_{count}, but not in L_{reg}. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.
- Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.