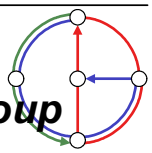
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing Group



WS 2006/2007 Dr. C. Stamm / Roland Flury / Stefan Schmid

Discrete Event Systems
Exercise 7

1 Dolce Vita in Rome

In order to relax a little bit from the busy life at ETH, Hector and his girlfriend Rachel decide to spend the weekend in Rome. Besides the cultural attractions, Hector and Rachel are also interested in the great choice of ice cream shops (*gelaterie*) which Rome offers.

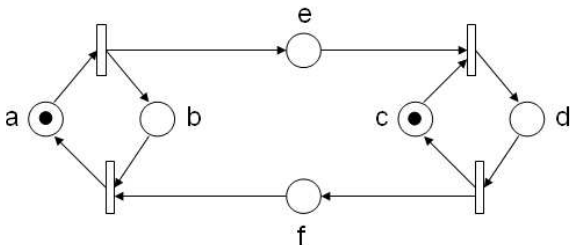
During their strolls through Rome, the two students encounter n gelaterie. Assume that these ice cream shops can be ranked uniquely according to their attraction, that is, for any two given shops, Hector and Rachel have a clear preference. For instance, the attraction may be a function of the price of the ice cream, quality, atmosphere of the shop, etc.

Since it's too expensive to eat ice cream on every occasion, the two students apply the following strategy: Whenever a shop i is more attractive than the shops 1 to $i-1$ which they have encountered so far, they buy an ice cream.

Assume that the ice cream shops appear in a random order, i.e., any one of the first i shops is equally likely to be the best so far. How many ice creams do Hector and Rachel consume during the weekend?

2 Token Game

In this exercise you are asked to study the dynamics of the following petri net with the start marking:



a) Is there a reachable marking where both places a and b have a token, i.e., where $M(\text{a}) \geq 1 \wedge M(\text{d}) \geq 1$ holds? Explain your decision.

b) Compute all reachable markings of the system or prove that there are infinitely many markings.

3 A Candlelight Dinner

Alice invites Bob for dinner. Unfortunately, in Alice' apartment-sharing community there is little cutlery.

a) As a starter, Alice prepared soup. There is only one spoon, and hence only one person can eat at any time. However, Alice and Bob are more interested in each other rather than in the food and are in no hurry. Therefore, whenever one of them has eaten some soup, the spoon is put back onto the table, and Alice and Bob have a little chat. At some later time, someone picks up the spoon again and eats some more. And so on.

(i) Model the situation using a petri net.

(ii) Prove that for your petri net, it holds that there is always at most one person having the spoon.

(iii) How would you change your net if Alice and Bob strictly alternated in eating?

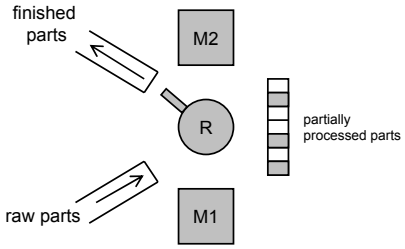
b) Additionally to the spoon, Alice finds a fork in the kitchen. As a second plate, they will have spaghetti and vegetables. "Of course", to eat spaghetti, one needs both a fork and a spoon. The vegetables on the other hand can be eaten either with a spoon or a fork (not both). Again, Alice and Bob are in no hurry and talk after each bite. At some time, someone takes the spoon and the fork and eats some spaghetti, or someone takes either the spoon or the fork and eats some vegetables. Model this situation using a petri net!

c) Assume there is a second fork. How could you change your petri net from the previous task to model also this situation?

d) Back to the situation with one fork and one spoon. Surprisingly, Trudy—Alice' room mate—comes back from a party. She takes a yoghurt from the fridge and sits down at the table. As you mighty already expect, she also needs a spoon. Extend your petri net!

4 Producer-Consumer System under Mutual Exclusion

We consider a pipelined factory where raw parts are preprocessed by a machine M1, stored in a temporary buffer, and finally assembled by a second machine M2. There is a single robot R that moves the parts between the input line, M1, the buffer, M2 and the output line. The buffer can hold at most 7 preprocessed items.



a) Use a Petri net to describe this system!

b) Show that the obtained Petri net is live (L4-live) and bounded.

c) Extend the Petri net such that the output of M1 can be transferred directly to M2, without being stored in the buffer.