

Discrete Event Systems

Solution to Exercise 7

1 Dolce Vita in Rome

Consider the following indicator variable for shop i : X_i is 1 if Hector and Rachel buy ice cream at the i -th shop and 0 otherwise. Since the probability that the i -th shop is the best so far equals $1/i$, we have $E[X_i] = 1/i \cdot 1 = 1/i$.

The total number of ice creams can be expressed by

$$X := X_1 + X_2 + \dots + X_n.$$

Therefore

$$E[X] = E\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n E[X_i] = \sum_{i=1}^n 1/i = H_n.$$

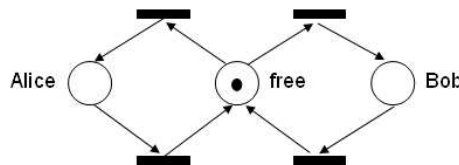
Thereby, H_n is the so-called *Harmonic Number*, which is roughly $H_n = \ln(n) + O(1)$. Thus, the two students roughly consume a logarithmic number of ice creams (in the total number of shops).

2 Token Game

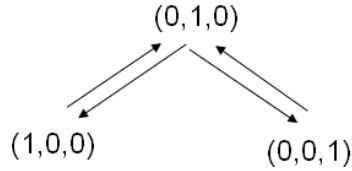
- a) In this example, it is easy to enumerate all possible markings in order to disprove the claim. As an alternative, note that the sum of tokens at the places a, e, d, and f is always at most 1: place e gets only a token if place a loses a token, place d gets only a token if place e loses a token, and similarly for places f and d and a and f.
- b) Starting with [a,c] and performing all possible transitions yields the following possible markings: {[a,c],[b,e,c],[d,b],[b,f,c]}. The number of markings is clearly finite.

3 A Candlelight Dinner

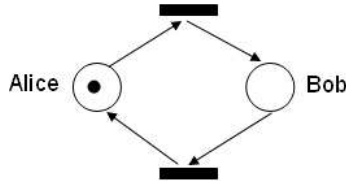
- a) (i) The following figure shows a possible solution with three places: Either Alice has the spoon or Bob, or the spoon lies on the table.



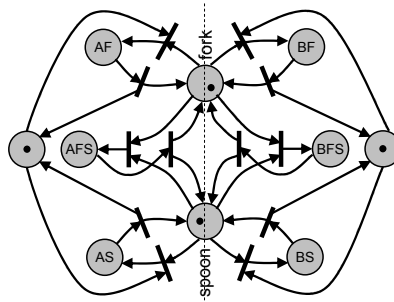
(ii) The claim follows directly from the finite state transition graph:



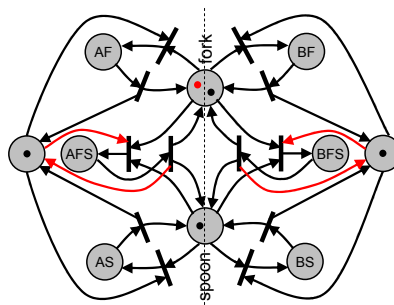
(iii) If Alice and Bob strictly alternate, the petri net looks as follows:



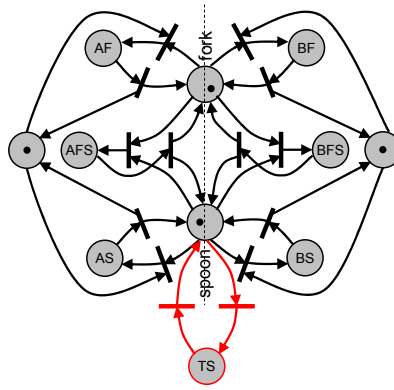
b) The situation could be modelled as shown in the following figure. Note that the petri net is symmetric, since Alice (petri net on the left) and Bob (petri net on the right) behave in the same way. Thereby, AF denotes that Alice has the fork and eats veggies, AS denotes that Alice has the spoon and eats veggies, and the place AFS denotes that Alice uses both fork and spoon to eat spaghetti. BF, BS, and BFS are the corresponding places of Bob. Finally, the place “fork” (or “spoon”) means that the fork (or spoon) is free.



c) The additional fork can be taken into account by adding a token to the place “fork free.” However, we have to make sure that neither Alice nor Bob has two forks at the same time. This requires two additional transitions as shown in the figure.



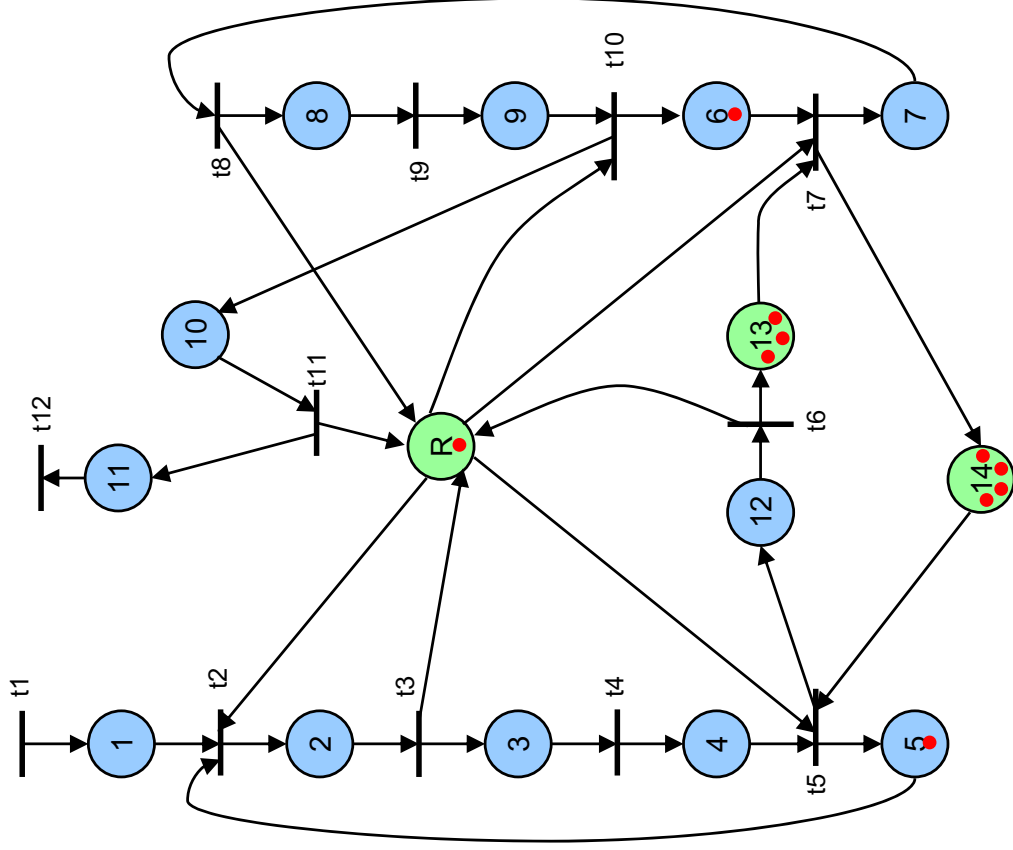
d) The petri net now looks as follows:



4 Producer-Consumer System under Mutual Exclusion

The petri nets are attached at the end of this document. Please note the tables describing the places and transitions of the nets.

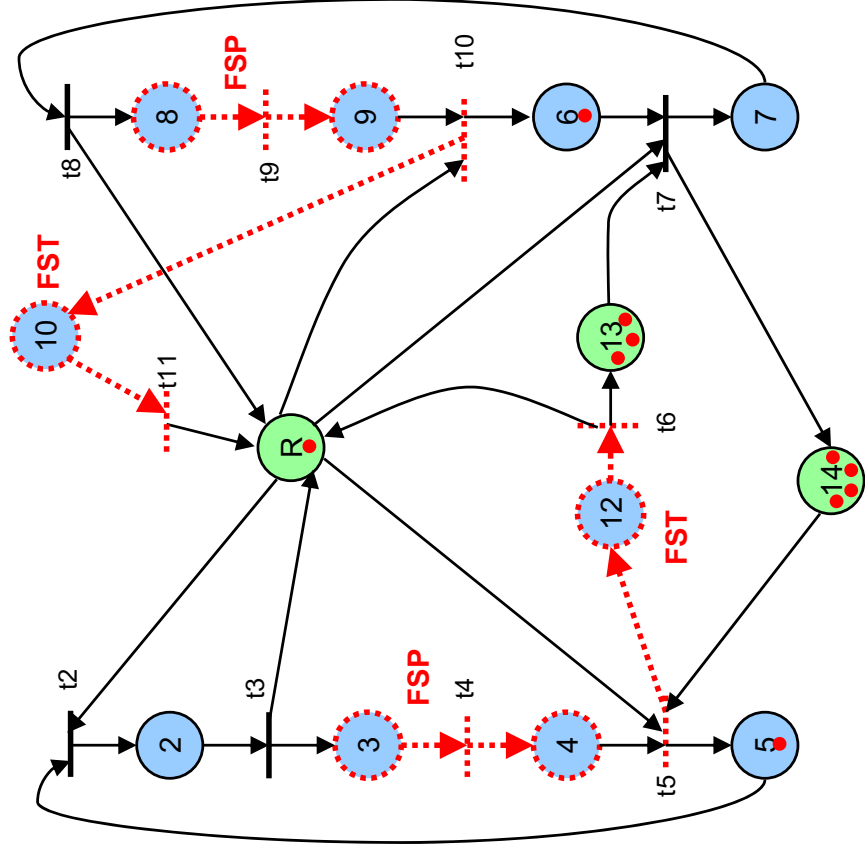
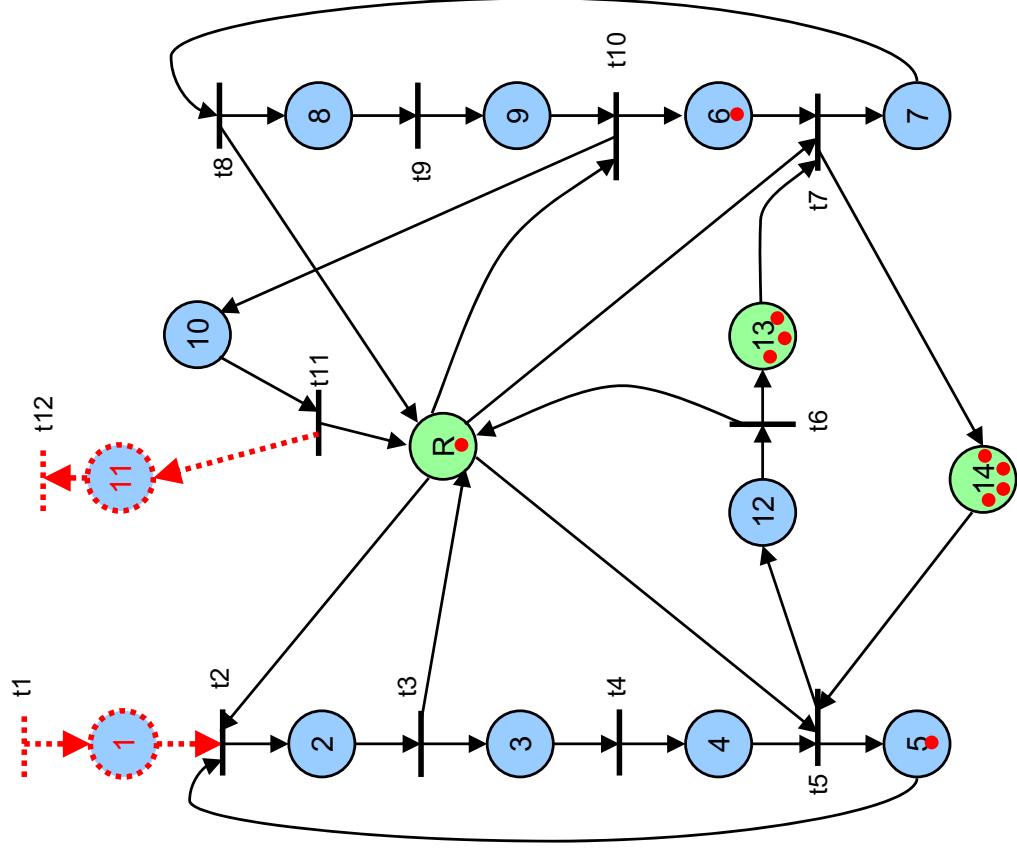
a) Draw the Petri net

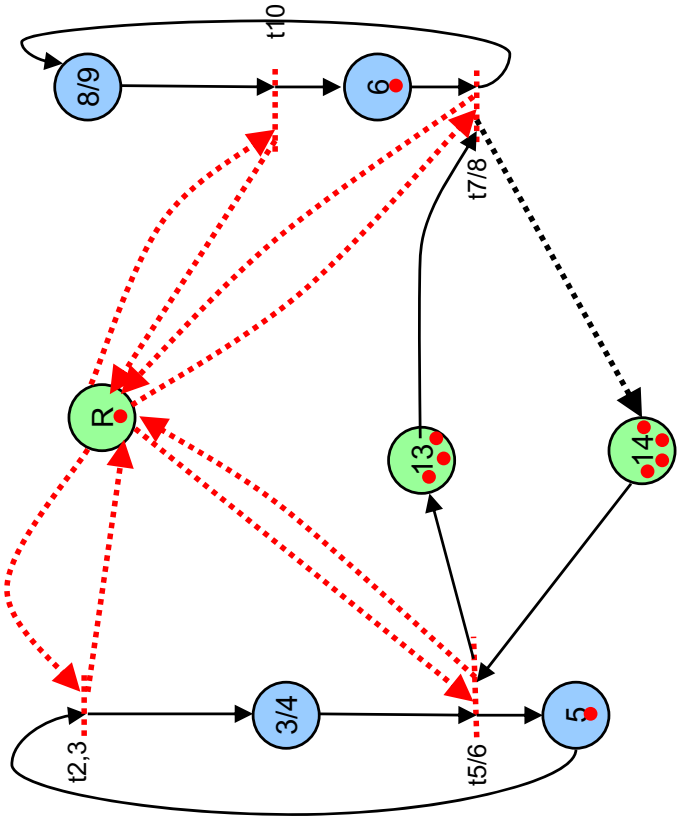
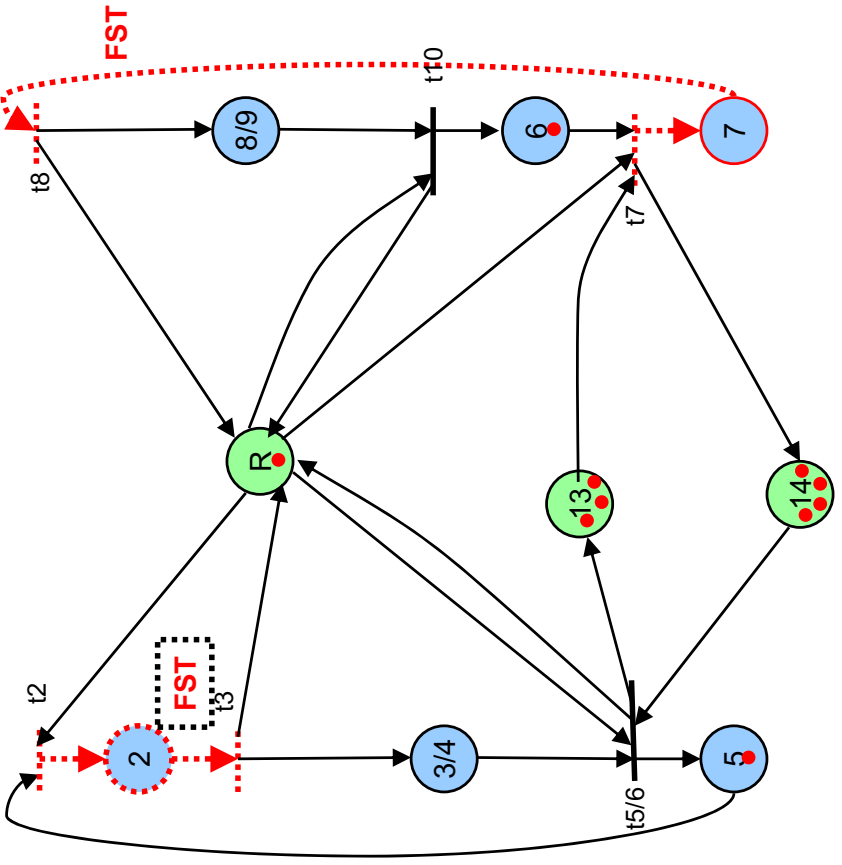


- t1 Raw parts arrive
- t2 Robot picks up raw parts
- t3 ... and delivers them to M1
- t4 M1 preprocesses raw parts
- t5 Robot picks up preproc. product from M1
- t6 ... and puts it into the buffer
- t7 Robot picks preproc. product from the buffer
- t8 ... and delivers it to M2
- t9 M2 finishes the product
- t10 Robot picks finished product from M2
- t11 ... and puts it to output line
- t12 Output line consumes product

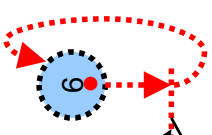
- 1 Models input line, each token represents the raw parts for one item
- 2 The robot holds the raw parts or one item and moves them to M1
- 3 M1 has the raw parts
- 4 M1 has finished preprocessing the raw parts
- 5 M1 is waiting for new raw parts
- 6 M2 is waiting for preprocessed parts from the buffer
- 7 The robot holds a preprocessed item retrieved from the buffer
- 8 M2 has a preprocessed item
- 9 M2 has finished producing the item
- 10 The robot holds the finished item and moves it to the output line
- 11 The output line holds the finished products until they're consumed (sink)
- 12 The robot holds a preprocessed item and moves it from M1 to the buffer
- 13 The buffer, holding the preprocessed items
- 14 The counter part to the buffer, ensuring its capacity constraints
- R The robot, under mutual exclusion for all tasks

b) Show that the obtained net is life and bounded

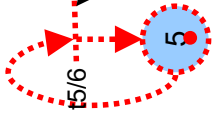




ESP



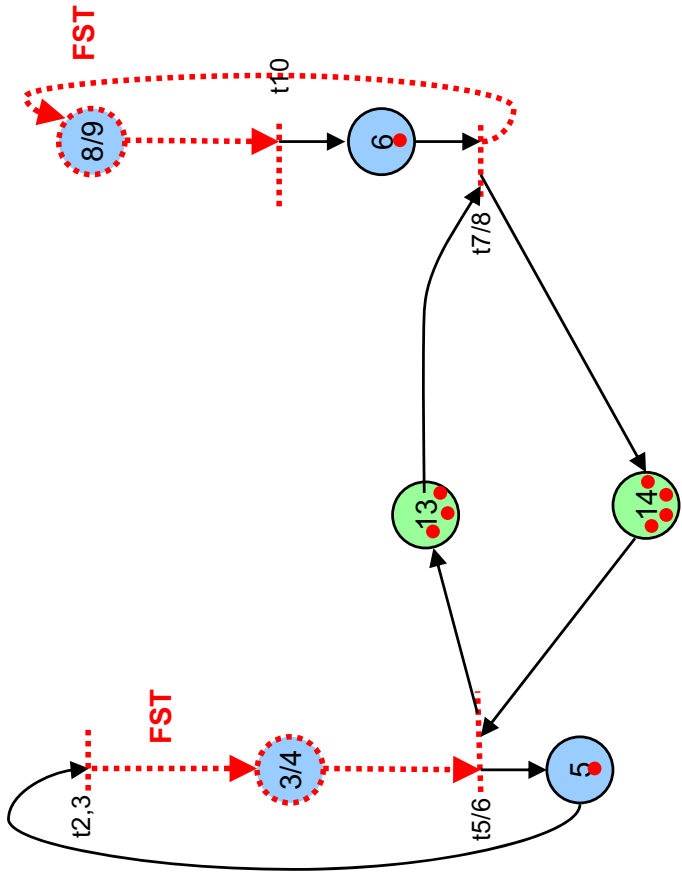
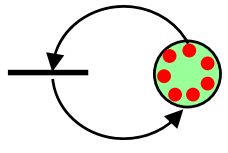
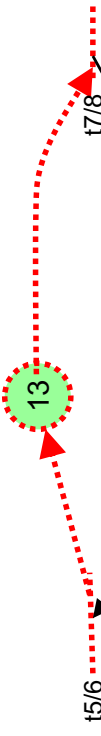
ESP



Fire 3 times



FST



c) Bypassing the buffer

